Progress in Quantum Electronics 55 (2017) 74–87

Contents lists available at ScienceDirect

Progress in Quantum Electronics

journal homepage: www.elsevier.com/locate/pqe

Review

Dressed photons in a new paradigm of off-shell quantum fields

Hirofumi Sakuma [a,*], Izumi Ojima [a], Motoichi Ohtsu [a,b]

[a] Research Origin for Dressed Photon, c/o Yokohama Technology Center, NICHIA Corporation, 3-13-19 Moriya-cho Kanagawa-ku, Yokohama-shi, 221-0022, Japan

[b] Institute of Engineering Innovation, School of Engineering, The University of Tokyo, 2-11-16 Yayoi Bunkyo-ku, Tokyo, 113-8656, Japan

ARTICLE INFO

2010 MSC:
00-01
99-00

Keywords:
Dressed photon
Off-shell
Energy-momentum support
Clebsch parameterization
Longitudinal mode

ABSTRACT

This article reviews recent progress in theoretical studies of dressed photons. For providing concrete physical images of dressed photons, several experimental studies are demonstrated. They are applications of dressed photons to novel optical functional devices, nano-fabrication technologies, energy conversion technologies, and photon breeding devices. After these experimental demonstrations, as the main part of this review, quantum-field theoretical formulation of dressed photons is attempted in use of the newly introduced Clebsch-dual variable of electromagnetic field. The reason for introducing the new formulation will be explained in the final section from the viewpoint to exhibit the contrast between free and interacting quantum fields in regard to their energy-momentum supports which are seldom touched upon (or forgotten) in the common physical discussions about quantum fields.

1. Introduction

The aim of the present paper is to clarify the conceptual and mathematical basis of the dynamic behaviors of *dressed photons* discovered by M. Ohtsu, one of the co-authors of this article, from the viewpoint of the duality relations between the usual free photon field and its "*Clebsch-dual* field", the latter of which is to be explained in what follows.

In the context of quantum field theory (QFT), the familiar common language is based on the particle pictures in terms of the creation $a_k^\dagger$ and annihilation a_l operators, $[a_l, a_k^\dagger]_\mp = \delta_{lk}$, acting on a Fock space constructed on the vacuum state $|0\rangle$ as the (symmetric or anti-symmetric) tensor powers $V = \underset{n\geq 0}{\oplus} V_n$, $V_0 = \mathbb{C}|0\rangle$, $V_n := V_1^{\otimes n}$ (: symmetric or anti-symmetric tensor product) of 1-particle states $V_1 := Lin\{a_k^\dagger\big|0\rangle; k : \text{momentum}\}$, $V_n = Lin\{a_{k_1}^\dagger a_{k_2}^\dagger \cdots a_{k_n}^\dagger\big|0\rangle; k_1,\ k_2,\ \cdots,\ k_n\}$. While this language is very convenient in treating many-body problems in both contexts of relativistic particle physics and of non-relativistic solid state physics, its serious pitfall lies in its "forgetfulness" about the strict restriction on its applicability in physical nature! Since the above simple-minded interpretation of quantum fields in terms of creation and annihilation operators is meaningful *only for free fields without interactions*, the validity of this vocabulary is at most for the *asymptotic fields and states* which describe quantum fields approximately only in the remote past or future regimes, respectively, before or after the actual scattering processes. The serious gap between interacting and free quantum fields in terms of Fock space language can clearly be seen in their totally different *energy-momentum spectral supports*, the latter being restricted to the mass hyperboloid $p^2 := p_\mu p^\mu = m^2$ but the former extending over the whole p-space(!!) according to a general result due to the axiomatic QFT

* Corresponding author.
E-mail address: sakuma@rodrep.or.jp (H. Sakuma).
URL: http://rodrep.or.jp/

http://dx.doi.org/10.1016/j.pquantelec.2017.07.006

Available online 27 July 2017

(as a mathematical formulation of quantum fields) [1].

The typical examples of this sort in the relativistic QFT on the vacuum are given by *intermediate states* between in-coming and outgoing states of scattering processes in particle physics, which are described in terms of *off-shell* particles and which cannot be identified with any on-shell free particles described by creation/annihilation operators of asymptotic fields. In solid state physics described by quantum statistical mechanics, another typical non-applicability domain of particle pictures can also be found in the well-known phenomena of *phase transitions* caused by the instability between two stable thermodynamic phases with different particle pictures, through which the same physical system undergoes drastic changes in material configurations: for instance, a phase transition from the *normal* phase with non-vanishing electric resistance to the *superconducting* one characterized by the vanishing resistance. These examples clarify that the *on-shell states* (abbreviation of "states on-the-mass shell") described by the creation and annihilation operators are not eternal invariants of a system but *variables* highly dependent upon the environments in which the relevant physical system is placed. When certain continuous symmetry present in one phase is broken in the other, there arise some *massless* excitation modes in the broken phase, according to a general theorem called Nambu-Goldstone theorem, as is the case of the phase transition from the normal one with unbroken electromagnetic $U(1)$ to the superconducting one with broken $U(1)$ symmetry.

Furthermore, the commitment of electromagnetic field (quantum and/or classical) causes more complications as will be discussed, like the case of *infrared divergences*, where "infrared soft photons" carrying low energies cause serious divergences through the coupling with the "on-shell states" of charged particles. What has been known as a solution to these infrared divergences is the mechanism to generate non-vanishing mass of photons due to the accumulated contacts between charged particles and soft photons which usually take place in continuous media: this kind of coupling mechanism has been found effective [2] in the mechanism involving the polaritons whose non-vanishing mass is directly related with the refractive index of the medium. Namely, the problem is solved in this case by a shift of the mass position of on-shell states, which defines new quasi-particles.

In the phenomena associated with dressed photons (to be abbreviated as DPs), however, this sort of quasi-particle pictures are not sufficient for explaining their dynamic behaviors, because of the strong localization of the associated creation and annihilation processes. To explain the characteristic dynamic behaviors of DPs, therefore, we need to incorporate the effects of interactions in a satisfactory way. Because of the above-mentioned serious differences between interacting and free fields with respect to their support properties in energy-momentum p_μ, this task seems to be quite non-trivial! For this purpose, our strategy is just to look for a free field $S_{\mu\nu}$, which resembles the free electromagnetic field $F_{\mu\nu}$ but whose p_μ-support covers all the spacelike p_μ's complementary to that $p^2 \geq 0$ of $F_{\mu\nu}$. If we succeed in formulating such $S_{\mu\nu}$, then the joint p_μ-support of $F_{\mu\nu}$ and $S_{\mu\nu}$ covers the whole p-space! Thus, we can mimic the essential features of interacting electromagnetic field by the combined use of two free fields $F_{\mu\nu}$ and $S_{\mu\nu}$. To achieve this task, a theoretical reformulation of electromagnetic theory is necessary as presented here from the viewpoint of *Clebsch variables*; corresponding to the free field strength $F_{\mu\nu}$ supported by lightlike energy-momentum $p^2 = 0$, its "*Clebsch-dual*" $S_{\mu\nu}$ can be constructed whose energy-momentum support covers the spacelike region $p^2 < 0$.

To understand the roles of energy-momentum supports in the present context, it may be useful to examine some example cases. For the simplest case, it is convenient to consider a relativistic process of two-body scattering involving four energy-momentum vectors, p_1, p_2 for incoming two particles and p_3, p_4 for outgoing two, in terms of which variable s, t and u are defined by $s := (p_1 + p_2)^2 = (p_3 + p_4)^2$, $t := (p_3 - p_1)^2 = (p_2 - p_4)^2$, $u := (p_4 - p_1)^2 = (p_2 - p_3)^2$ (: so-called Mandelstam variables). s represents essentially the square of conserved energy in the system of center of mass of incoming two particles, t the square of energy transfer from the incoming particle 1 to the outgoing 3 and u corresponds to t in the crossed channel. In these variables, the process is described by *positive* variable s and by *negative* ones t and u, the latter of which are closely related with the *interaction or potential terms* between two particles. The first one is usually called s-channel, and the second and the third ones t- and u-channels. By taking such parallelisms as $F_{\mu\nu} \leftrightarrow s$-channel and $S_{\mu\nu} \leftrightarrow t$- and u-channels, one can get some instructive images about "*Clebsch-dual*" $S_{\mu\nu}$.

The organization of this paper is as follows. After reviewing in Section 2, the past and present status of the theoretical description of phenomena involving DPs, we try in Section 3 to show recent progress in experimental studies of DPs [3]. Here, we should point out that there still remain problems to be solved for gaining a deeper understanding of DPs and exploring more applications. For the purpose of solving these problems, we present the Clebsch dual field of electromagnetic field in Section 4. The final section, Section 5, is devoted to the discussion for summarizing the presented concepts and the context formed to clarify the meaning of the key concept of DPs.

2. Dressed photon as a physical picture of an off-shell photon

As mentioned in the previous section, the concept of elementary excitations [4] is working effectively everywhere in solid state physics: namely, excited states of a many-body system can be described in terms of a collection of certain fundamental excited modes called elementary excitations or "quasi-particles" in short. An exciton as a well-known example represents a quasi-particle related to an electron-hole pair in a solid. The interaction between a photon and an exciton forms a new steady state which also represents a quasi-particle called an exciton-polariton. Its dispersion relation between the wavenumber and energy of the exciton-polaritons in macroscopic space is described by curves A and B in Fig. 1. Note that other kinds of quasi-particles also show dispersion relations similar to these curves.

In this figure, we note the presence of a wide space around the curves A and B, as shown by the green shaded rectangle. A quasi-particle is created also in this space whose large size gives the following characteristic features to the created quasi-particle:

(1) As represented by the horizontal double-pointed gray arrow, the wavenumber k of this quasi-particle spans a wide range. This also means that its uncertainty Δk is large, which is related to the non-conservation of momentum $\hbar k$ of the quasi-particle (where

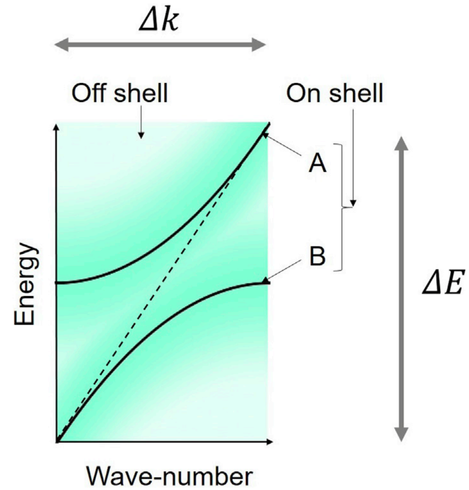

Fig. 1. Dispersion relation between the wave-number ***k*** **and the energy** *E*. The curves A and B represent the dispersion relation of the exciton-polariton. The broken line is for light in free space. The green shaded rectangle is for the dressed photon.

$\hbar = h/2\pi$ and h is the Planck constant) involving a large number of normal modes. At the same time, this implies, according to the Heisenberg uncertainty relation $\Delta(\hbar k)\cdot\Delta x \geq \hbar$, that the uncertainty Δx of position x of the quasi-particle is small, and hence, the size of this quasi-particle is very small.

(2) As is represented by the vertical double-pointed gray arrow, the energy E of this quasi-particle spans a wide range, and hence, its uncertainty ΔE is large, which implies the non-conservation of the energy. Owing to the Heisenberg uncertainty relation $\Delta E\cdot\Delta t \geq \hbar$ with large uncertainty ΔE, the time uncertainty Δt is small. Therefore, this quasi-particle is created and subsequently annihilated within a short time.

The small size in the first feature is due to the small scale of such materials as nanomaterial to create the quasi-particle, where a part of the field generated by this quasi-particle penetrates through the surface of the nanomaterial to the outside. This leaking field has been called optical near field [5]. The second feature of the short lifetime suggests the interpretation of the quasi-particle as a sort of virtual photon. The concept of a DP provides a unified physical picture for describing these two features consistently, including both the large number of normal modes and the spectral sideband features of (1) and (2).

To compare these features with the exciton-polariton represented by the curves A and B in Fig. 1, we note that the latter one represents a quasi-particle in a macroscopic material with the size much larger than the wavelength of light. This quasi-particle propagates into a wide space as a real photon; its momentum and energy are conserved ($\Delta k = 0$, $\Delta E = 0$). The behavior and properties of this real photon have long been studied in the traditional optical science and technology. The situation corresponds to a physical system described by states "on-the-mass shell" (or, on-shell states, in short) in QFT [6–8].

In contrast, the green shaded rectangle corresponds to the situations with "off-shell" photons. While real on-shell photons conserve their momentum and energy, off-shell virtual photons do not. Light-matter couplings in nanometric space can exhibit characteristic distributions in the values of the physical quantities of nanomaterials, such as momentum and energy. To study the off-shell virtual photons, we note that there are such unsolved problems and untouched subjects in analyzing these distributions as

(I) QFT is required to describe the numbers of photons and electron-hole pairs in a many-particle system, which can vary as a result of light-matter couplings in a nanometric space. For this description, creation and annihilation operators have to be defined by quantizing the light and elementary particles. However, a serious problem for off-shell photon is that a virtual cavity cannot be defined in a sub-wavelength sized nanometric space, which makes it difficult to derive the Hamiltonian of the optical energy. In addition, the wavelength (wavenumber) of light and the photon momentum have large uncertainties due to the sub-wavelength size of the space under study.

(II) The off-shell photons are generated and localized on the surface of a nanomaterial under light irradiation. Furthermore, they are virtual photons mediating the interaction between the polarizations induced on the nanomaterial surface. For detecting these photons, therefore, another nanomaterial has to be placed in close proximity to convert them into real photons via multiple scattering of the virtual photons. This conversion makes it possible to detect the virtual photons through the detection of the scattered light in the far field region. Here, the first and the second nanomaterials may be regarded as the source and detector of the virtual photons, respectively. In contrast to the conventional optical phenomena, however, the source and detector are not independent of each other, but are coupled via the virtual photons.

(III) Since the nanomaterials in an actual nanometric subsystem are always surrounded by a macroscopic subsystem composed of macroscopic materials and electromagnetic fields, the contribution from the macroscopic subsystem must be taken into account in analyzing the interactions between the nanomaterials for estimating the magnitude of the resulting energy transfer and dissipation in the nanometric subsystem.

For solving the above problems (I)-(III), novel theories have been developed and these theories have succeeded in providing a physical picture of the optical near fields and virtual photons consistently, which has been beyond the traditional scope of conventional classical and quantum optics designed only for the on-shell photons in the macroscopic space. Solutions to problems (I)-(III) are as follows:

(i) To solve problem (I), an infinite number of electromagnetic modes with infinite frequencies, polarization states, and energies are assumed, which correspond to the large number of normal modes and the spectral sideband features of (1) and (2) above. An infinite number of energy states are also assumed for the electrons and holes. On these assumptions, the total Hamiltonian is derived to define the creation and annihilation operators of quasi-particles so as to represent the light-matter interactions in the nanometric space [9]. Since these operators are given by the sum of the operators of photons and electron-hole pairs, this quasi-particle is called a DP as a photon dressed in the clothes of material energy of the electron-hole pairs [10]. The DP can also couple with multi-mode coherent phonons in the nanomaterial to create another quasi-particle, named the dressed-photon-phonon (DPP) [11].

(ii) To solve (II), the theoretical approach reviewed in (i) has been used for analyzing the interaction between the two nanomaterials in terms of the annihilation of a DP from the first nanomaterial and of its creation on the second nanomaterial. It has been confirmed by experimental and theoretical studies that the second particle exhibits a characteristic optical response if it absorbs the energy of the modulation sideband described in (i).

(iii) To solve (III), the virtual photon interaction between the nanomaterials in the nanometric subsystem has been analyzed by renormalizing the effects originating from the macroscopic subsystem in a consistent and systematic way [12]. As a result, the spatial distribution of the virtual photon interaction energy can be derived and expressed in terms of a Yukawa potential, which also makes it possible to describe the interaction between the two nanomaterials mediated by the DP (DP-mediated interaction) [13]. This function quantitatively shows that the interaction range is equivalent to the size of the nanomaterial and does not depend on the wavelength of the incident propagating light. Starting from this size-dependent interaction range, novel optical response characteristics have been determined: since the DP is localized in nanometric space, the conventional long-wavelength approximation valid for the light-matter interactions in macroscopic space is not applicable to the DP-mediated interaction. As a result, a transition forbidden via electric dipole becomes an allowed one. Furthermore, size-dependent resonance and spatial hierarchy in the DP-mediated interaction has been found [14].

3. Applications of dressed photons

In this section, recent progress in experimental studies of DP is reviewed. There are four examples of applications that have been developed by using the intrinsic features of the DP-mediated interaction and the resulting DP energy transfer.

(1) Optical functional devices: These devices have been named DP devices by using semiconductor nanomaterials. They enable the transmission and readout of optical signals by the energy transfer and subsequent dissipation of the DP energy [15]. Examples of DP devices developed so far are as follows: a logic gate device for controlling optical signals [16], an energy transmitter for transmitting optical signals between DP devices [17], and an input interface device for converting an incident on-shell photon to a DP [18]. Practical NOT gate and AND gate devices that operate at room temperature have also been fabricated by using InAs nanomaterials [19].

(2) Nano-fabrication technologies: By photochemical etching, bumps on a rough material surface are autonomously removed by using chemically radical atoms. These atoms are created by photo-dissociating gaseous molecules due to DPP energy transfer from the apex of the bump to the molecules under visible light irradiation [20]. Using this method, the surfaces of glass substrates have been smoothed for use as high-power laser mirrors [21], for magnetic storage memory disks [22], for EUV masks [23], and for side walls of densely aligned corrugations of a diffraction grating [24]. This method has been applied to other materials, such as plastic PMMA [25], crystalline GaN [26], and crystalline diamond [27].

(3) Energy conversion:

(3-1) Optical-to-optical energy conversion: Near infrared light has been converted to visible light by using energy up-conversion in the process of DPP energy transfer between organic dye particles. Red, green, and blue light has been emitted from dye particles by irradiating them with 0.8–1.3 μm wavelength infrared light [28,29].

(3-2) Optical-to-electrical energy conversion: A photovoltaic device using an organic P3HT film has been developed [30]. The electrode surface conformation of this device was autonomously modified by using novel DPP-assisted deposition of silver particles for efficient DPP generation. A Si photodiode has been also developed, in which the spatial distribution of doped boron (B) atoms was autonomously modified by a novel DPP-assisted annealing method [31]. In these two devices, a phenomenon, "photon breeding", (refer to (4) below) has been confirmed. Furthermore, in the case of the Si photodiode, an optical amplification capability was confirmed, which is due to stimulated emission triggered by the DP.

(4) Photon breeding devices: They are novel LEDs fabricated by using bulk crystals [60–62, Springer Si LED&LD] [32–35] of indirect-transition-type Si semiconductors. The autonomous modification method reviewed in (3-2) above was also used, resulting in efficient momentum transfer between electrons and phonons in Si. Besides Si, an indirect transition-type GaP semiconductor has

been used for fabricating an LED emitting yellow-green light [36,37]. An indirect transition-type SiC semiconductor has also been used for fabricating LED emitting blue-violet light [38], ultraviolet light [39], and white light [40]. In addition to LEDs, an optical and electrical relaxation oscillator [41] and a near infrared laser have been realized by using crystalline Si [42–44]. Furthermore, an LED [45], a light polarization rotator [46], and a light beam deflector [47] have been developed by using ZnO crystal. A light polarization rotator using SiC crystal has been also developed [48].

For simple demonstration, Fig. 2(a) shows the cross-sectional structure of the Si crystal used as the laser medium (1 mm width, 150 μm thickness) [49,50]. The length of the crystal is 15 mm, as seen in the photograph in Fig. 2(b). Closed squares in Fig. 2(c) mean the measured relation between the injection current density J and the optical output power P_{out} of 1.3 μm-wavelength, emitted from one output facet of the fabricated device. The threshold current density J_{th} is 60 A/cm^2. The value of P_{out} takes a maximum value of 13 W at $J = 100A/cm^2$. This value is more than 10^2-times higher than that of a conventional double heterojunction-structured InGaAsP/InP laser.

Remaining part of this section is devoted to review how to fabricate and operate the photon breeding devices because this review demonstrates characteristic features of DPs and how to gain a deeper understanding of DPs, which will be discussed in next sections.

To realize a device by using a Si bulk crystal, DPPs are used twice: first for device fabrication, and next for device operation.

(4-1) For device fabrication, a p-n homojunction formed in the Si crystal is annealed, via Joule-heat produced by current injection, in order to diffuse B atoms (the p-type dopant). During the annealing, the Si crystal surface is irradiated with light (Fig. 3(a)) to create DPPs on the B atom surface. Driven by the created DPPs, electron-hole recombination takes place, emitting light. Since the energy of the emitted light dissipates from the Si crystal, the efficiency of the Joule-heating decreases. As a result, a characteristic spatial distribution of B atoms is realized, which depends on the created DPP energy. This novel annealing is called DPP-assisted annealing. It has been experimentally confirmed that, in this spatial distribution, neighboring B atoms form a pair, and the resultant B atom-pair orients in a specific direction to efficiently create localized phonons [51].

(4-2) For the operation of the fabricated Si-LED, the light irradiation is not required any more; it is used only during the DPP-assisted annealing. Only forward current that is much lower than that used for annealing is injected, as is the case of the conventional LED operation. By this forward current, an electron is injected into the conduction band at the p-n homojunction and creates a photon by spontaneous emission even though its probability is very low. However, once this photon is created, it subsequently creates a DPP on the surface of the B atom at the p-n homojunction, and this DPP interacts with another electron in the conduction band to exchange momentum so that a secondary photon is created. By repeating these momentum exchange and photon creation processes, the emitted light intensity is amplified and reaches a stationary value within a short period, so that light with a sufficiently high intensity is emitted from the p-n homojunction.

It should be noted that photon breeding occurs during device operation [52]. As a result, the photon energy of the emitted light is equal to the photon energy $h\nu_{annsal}$ of the light irradiated during the annealing (Fig. 3(b)). This is in contrast to a conventional device, where the photon energy of the emitted light is determined by the bandgap energy E_g of the semiconductor material used. This is also because the difference between $h\nu_{annsal}$ and E_g is compensated for by the energy of the created phonons. This means that the photon energy of the light emitted from the device is identical to $h\nu_{annsal}$. This is because the spatial distribution of the B atoms has been controlled by the light irradiated during the DPP-assisted annealing, enabling most efficient stimulated emission and spontaneous emission of photons with identical photon energy. In other words, the light irradiated during the DPP-assisted annealing serves as a "breeder" that creates photons with an energy equivalent to $h\nu_{annsal}$. This is the reason why this novel phenomenon is named photon

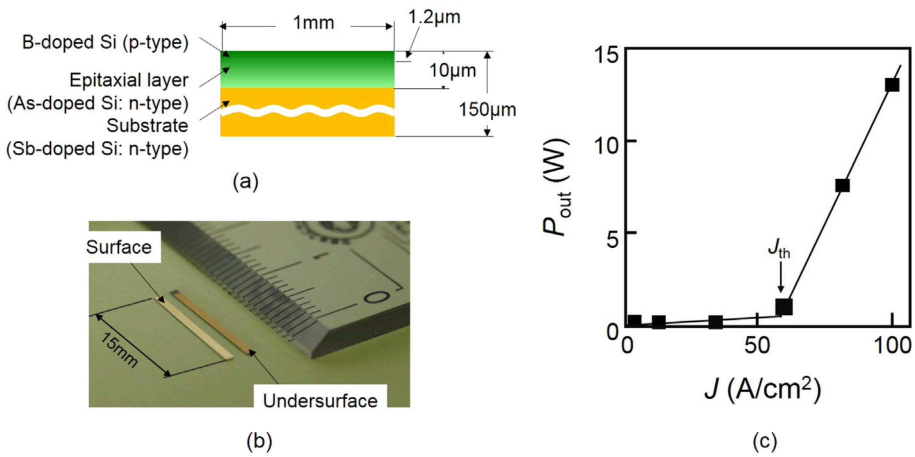

Fig. 2. A near infrared laser. (a) Cross-sectional profile of the Si crystal. (b) Photograph of the laser. (c) Measured relation between the injection current density, $J(A/cm^2)$, and the optical output power, $P_{out}(W)$, emitted from one output facet of the Si crystal.

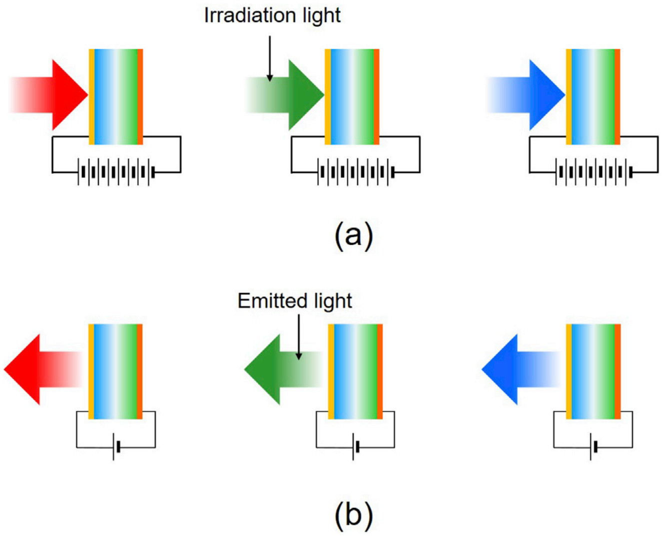

Fig. 3. Fabrication (a) and operation (b) of a photon breeding device.

breeding with respect to photon energy.

Photon breeding has been observed not only for the photon energy but also for the photon spin [51]. For example, linearly polarized light is emitted from the LED if it is fabricated by irradiating linearly polarized light during the annealing step.

At the end of this section, it should be pointed out that there still remain problems to be solved for gaining a deeper understanding of DPs and exploring more applications. These include: (1) Improving the accuracy of the physical picture of the quasi-particle representing the coupled state of a photon, an electron, and a phonon in a nanometric space. (2) Detailed description of energy transfer and dissipation between nanomaterials, mediated by DPs or DPPs. (3) Detailed description of phonon-assisted light-matter interactions in nanometric space. (4) Detailed description of the physical origins of autonomy and hierarchy.

For more advanced methods that will allow us to see the DP and to describe its origin, further studies on the energy transfer from the nano- to macro-systems are required. In particular, as a comment on problem (4) above, the physics of this complex system should be developed on the basis of micro-macro duality in quantum theory [53], which facilitates the formation of micro-macro composite systems consisting of the object physical systems and the probe systems [2]. The specific features that originate from the inherent hierarchy of DPs will be smoothly and conveniently described in this framework with the aid of category-theoretical concepts [54].

4. Novel theoretical attempt on modeling of virtual photon

4.1. Polarization and so far unnoticed duality of electromagnetic field

Since the concept of virtual photons (similarly to that of other particles) has emerged from the perturbative calculations in QFT, its definition may look somewhat loose compared to that of real photons. The present situation lacking a clear picture or clear understanding of DP as a form of virtual photons seems to reflect this vagueness in its definition. Since real and virtual photons are physical entities responsible for the dynamical processes of charged particles, however, they both should be treated on the equal footing.

As we have seen in sections 2 and 3, one of the essential aspects of DP dynamics is its active coupling with other fields generated by materials ubiquitously present in the environment. This feature, however, need not necessarily lead to the absence of a clear physical picture of DP since not all the properties but the intrinsic ones of photons must be preserved in such processes of interactions and since the ground for calling them "photon" would otherwise be lost. An instructive example of such a process is a well-documented mass-acquiring mechanism of a photon as a massless gauge boson in which polarization and longitudinal mode play a significant role.

Motivated by this perspective, we exhibit new aspects about the mutual relations (=duality) between what is dominantly visible and what is invisible in a given domain, from a flexible viewpoint that such mutual relations can easily be changed through a shift from one situation to another. In the specific context of the mathematical description of virtual photons, we show the presence of a duality of electromagnetic field hidden so far. Along this line, we can benefit from the guiding principle of *quantum-classical correspondence* whose importance has been recognized anew in the context of Micro-Macro Duality due to one of the authors [55], aiming at overcoming the drawback of disconnected relations between micro and macro physics caused inadvertently through many twists and turns in the history of quantum mechanics.

4.2. Clebsch parameterized vortex model

It is obvious that an important missing factor in the particle model of geometric optics is the "spin" of a particle. Since we confine ourselves to the classical physics here, by "spin" we mean rotational field associated with the velocity of a given particle, *i.e.*, vorticity in hydrodynamic terminology. Since the actual spin of photon is closely related to the polarization of associated wave representation, it is relevant to focus on this quantity here. In geometric optics, a light ray is represented as a null geodesic (line) in a four dimensional (4d) pseudo Riemannian manifold ***M***. A geodesic is characterized by the property that adjacent tangential vectors on it are connected by parallel displacement, so that velocity field U_μ of (the classical) light particle tangential to a geodesic assumes the form of

$$(\nabla_U U)_\mu = U^\nu \nabla_\nu U_\mu = U^\nu \left(\nabla_\nu U_\mu - \nabla_\mu U_\nu\right) + \nabla_\mu (U^\nu U_\nu/2) = 0, \tag{1}$$

where ∇_X denotes the covariant derivative along a vector field $X = X^\mu \partial_\mu$ associated with the Levi-Civita connection. Since the Levi-Civita connection is torsion-free and since a given geodesic field is null, Eq. (1) is seen to reduce to a simpler form as

$$U^\nu \nabla_\nu U_\mu = U^\nu \left(\partial_\nu U_\mu - \partial_\mu U_\nu\right) = U^\nu S_{\mu\nu} = 0, \tag{2}$$

where $S_{\mu\nu} \equiv \partial_\nu U_\mu - \partial_\mu U_\nu$ is a vorticity field derived from U_μ. Note that a spinless (irrotational) velocity field $U_\mu = \partial_\mu \phi$ automatically satisfies (2), which corresponds to a conventional model of geometric optics.

In order to have a spinning model of the classical "photon", let us consider a Clebsch parameterized (CP) velocity field U_μ [56] defined originally by

$$U_\mu \equiv \lambda \nabla_\mu \phi + \nabla_\mu \chi. \tag{3}$$

While the Clebsch parameterization is well known in the context of the Hamiltonian structure of barotropic fluid [57], to our best knowledge, a preliminary study [58] due to one of the authors' (H. S.) seems to be the only case of its application to a null geodesic field. Since we are concerned with rotational modes of U_μ, we consider in what follows a reduced form of (3), namely,

$$U_\mu = \lambda \nabla_\mu \phi. \tag{4}$$

Notice first that the reduced CP flow field is parameterized by two parameters, in which we must specify differential equations for ϕ and λ. Here, we assume that

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0; \quad \left(g^{\mu\nu} \nabla_\nu \phi \nabla_\mu \phi = 0\right), \tag{5}$$

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \lambda - (\kappa_0)^2 \lambda = 0; \quad \left(g^{\mu\nu} \nabla_\nu \lambda \nabla_\mu \lambda = -(\kappa_0)^2 \lambda^2\right), \tag{6}$$

with a certain real constant κ_0. The reason why we select a spacelike Klein-Gordon equation for Eq. (6) will be seen shortly. Since Eqs. (5) and (6) are scalar equations, we can impose a directional constraint on $\nabla_\mu \phi$ and $\nabla_\mu \lambda$. For notational simplicity, let us define covectors (abbreviation of covariant vectors) C_μ, L_μ and a simple bivector $S_{\mu\nu}$ as

$$C_\mu \equiv \nabla_\mu \phi; L_\mu \equiv \nabla_\mu \lambda; S_{\mu\nu} \equiv \nabla_\nu U_\mu - \nabla_\mu U_\nu = C_\mu L_\nu - L_\mu C_\nu. \tag{7}$$

By the last equation in Eq. (7) a skew-symmetric $S_{\mu\nu}$ is given by an exterior product of C_μ and L_μ, according to the definition of a simple bivector which plays an important role in our CP formulation. In our reduced CP, we adopt such a directional constraint as

$$C^\nu \nabla_\nu L_\mu = 0, \tag{8}$$

which simply says that L_μ is advected (or convected) by C^ν. Immediate consequences of this constraint are as follows:

$$L^\mu \left(C^\nu \nabla_\nu L_\mu\right) = 0; \Rightarrow C^\nu \nabla_\nu \left(L^\mu L_\mu\right) = 0; \tag{9}$$

$$\begin{aligned} C^\mu \left(C^\nu \nabla_\nu L_\mu\right) = 0; \Rightarrow & C^\nu \left[\nabla_\nu \left(C^\mu L_\mu\right) - L_\mu \nabla_\nu C^\mu\right] \\ &= C^\nu \nabla_\nu \left(C^\mu L_\mu\right) = 0. \end{aligned} \tag{10}$$

In deriving (10), we have used $C^\nu \nabla_\nu C^\mu = 0$, which says that C^ν itself satisfies the null geodesic equation. By (10), the following orthogonality can be assumed between the two vectors L^μ and C^μ as an initial condition for (8):

$$L_\nu C^\nu = 0. \tag{11}$$

Since any vector perpendicular to a given null vector is either the same null vector or a spacelike one, we can choose L_ν such that it is a spacelike vector satisfying (11), namely,

$$\rho \equiv -L^\nu L_\nu > 0. \tag{12}$$

With the orthogonality relation given in Eqs. (11) and (2) becomes

$$U^\nu \nabla_\nu U_\mu = S_{\mu\nu} (\lambda C^\nu) = \left(C_\mu L_\nu - L_\mu C_\nu\right)(\lambda C^\nu) = (L_\nu C^\nu) \lambda C_\mu = 0, \tag{13}$$

according to which CP flow represented by Eq. (4) satisfies a null geodesic equation. We note here the similarity between Eq. (13) and the following equation for the electromagnetic field:

$$F_{\mu\nu}P^{\nu} = 0, \tag{14}$$

where $F_{\mu\nu}$ and P^{ν} denote, respectively, the electromagnetic skew-symmetric tensor and Poynting 4-vector. The latter one is parallel to a null geodesic perpendicular to both the electric $\vec{E} = (F_{01}, F_{02}, F_{03})$ and the magnetic $\vec{H} = -(F_{23}, F_{31}, F_{12})$ vectors. The difference in form between Eqs. (13) and (14) is that the skew-symmetric tensor $F_{\mu\nu}$ is not defined as the curl of P_{μ} while $S_{\mu\nu}$ is derived by the curl of U_{μ}.

One of the direct consequences of Eq. (4) is the following identity:

$$D \equiv S_{01}S_{23} + S_{02}S_{31} + S_{03}S_{12} = 0, \tag{15}$$

where $D = Pf(S)$ is the Pfaffian of the anti-symmetric matrix $S_{\mu\nu}$: $D^2 = Det(S_{\mu\nu})$. Similarly to $\vec{E}\cdot\vec{H} = 0$ for the electromagnetic field $F_{\mu\nu}$, the orthogonality between (S_{01}, S_{02}, S_{03}) and (S_{23}, S_{31}, S_{12}) follows from the existence of non-zero vector U^{ν} satisfying the matrix Eq. (13) because of $Det(S_{\mu\nu}) = D^2$.

4.3. Lightlike field with an embedded classical wave-particle duality

Corresponding to the energy-momentum tensor T_{μ}^{ν} (in the mixed tensor form) associated with electromagnetic field given by

$$T_{\mu}^{\nu} = -F_{\mu\sigma}F^{\nu\sigma}, \tag{16}$$

we have the similar quantity $\widehat{T}_{\mu}^{\nu}$ for $S_{\mu\nu}$ defined by

$$\begin{aligned}\widehat{T}_{\mu}^{\nu} &= -S_{\mu\sigma}S^{\nu\sigma} = -\left(C_{\mu}L_{\sigma} - L_{\mu}C_{\sigma}\right)\left(C^{\nu}L^{\sigma} - L^{\nu}C^{\sigma}\right)\\ &= -(L_{\sigma}L^{\sigma})C_{\mu}C^{\nu} = \rho C_{\mu}C^{\nu},\end{aligned} \tag{17}$$

where use has been made of equations (5), (11) and (12). While C^{μ} is lightlike, (17) is identical in form to the energy-momentum tensor of freely moving (fluid) particles. Actually, from $C^{\nu}\nabla_{\nu}C_{\mu} = 0$, $\nabla_{\nu}C^{\nu} = 0$ and $C^{\nu}\nabla_{\nu}\rho = 0$, owing to (5), (9) and (12), we get

$$\nabla_{\nu}\widehat{T}_{\mu}^{\nu} = 0. \tag{18}$$

Since (17) has dual representations, (18) can also be expressed in terms of $S_{\mu\nu}$ as

$$\nabla_{\nu}\widehat{T}_{\mu}^{\nu} = -\nabla_{\nu}\left(S_{\mu\sigma}S^{\nu\sigma}\right) = -S_{\mu\sigma}\nabla_{\nu}S^{\nu\sigma} = 0. \tag{19}$$

In deriving the above equation, the well-known Bianchi identity $\nabla_{\nu}S_{\rho\sigma} + \nabla_{\rho}S_{\sigma\nu} + \nabla_{\sigma}S_{\nu\rho} = 0$ is used in combination with the condition $S_{\nu\rho}S^{\nu\rho} = 0$ corresponding to massless property of electromagnetic field. An intriguing consequence of the dual representation (17) is that we readily get the classical version of Einstein - de Broglie relation which cannot be recovered by the conventional classical electromagnetic theory. In fact, we have, on the basis of (17),

$$\widehat{T}_{0}^{\nu} = -S_{0\sigma}S^{\nu\sigma} = \rho C_0 C^{\nu}, \tag{20}$$

which is the Poynting four vector representing the flux of energy density. The hydrodynamic representation on r.h.s. readily allows us to regard this as the advection of energy density ρC_0 by a given "velocity" field C^{ν}. In addition, since ρ can be regarded as density, namely, the number of "molecules" per unit volume, the energy associated with one "molecule" is proportional to the frequency of ϕ field.

Going back to (19), we see that the corresponding quantity $\nabla_{\nu}T_{\mu}^{\nu} = -F_{\mu\sigma}\nabla_{\nu}F^{\nu\sigma}$ in Maxwell's theory of electromagnetism vanishes under the following condition of no electric current:

$$\nabla_{\nu}F^{\nu\sigma} = 0, \tag{21}$$

which yields the electromagnetic wave equation in the vacuum. Note, however, that $\nabla_{\nu}S^{\nu\sigma} = 0$ is sufficient for (19) but not necessary, and, in the case of CP flow under consideration, simple manipulations on (6) yields

$$\nabla_{\nu}S^{\nu\sigma} = -(\nabla_{\nu}L^{\nu})C^{\sigma} = -(\kappa_0)^2\lambda C^{\sigma} = -(\kappa_0)^2U^{\sigma}. \tag{22}$$

By virtue of (13), therefore, Eq. (19) holds whether $\nabla_{\nu}S^{\nu\sigma} = 0$ holds or not. Note that, because of $\nabla_{\nu}S^{\nu\sigma} = \nabla_{\nu}(\nabla^{\sigma}U^{\nu} - \nabla^{\nu}U^{\sigma})$, Eq. (22) can be rewritten as

$$\nabla_{\nu}(\nabla^{\nu}U^{\sigma} - \nabla^{\sigma}U^{\nu}) - (\kappa_0)^2U^{\sigma} = 0, \tag{23}$$

which can be regarded as "Proca equation" in our CP vortex dynamics. The original Proca equation for electromagnetic theory takes the form:

$$\nabla_{\nu}(\nabla^{\nu}A^{\sigma} - \nabla^{\sigma}A^{\nu}) + m^2A^{\sigma} = 0, \tag{24}$$

in terms of the electromagnetic vector potential A^μ and the mass m^2. Eq. (24) clearly shows that lightlike solutions to $\nabla_\nu(\nabla^\nu A^\sigma - \nabla^\sigma A^\nu) = 0$ are gauge invariant, while solutions to (24) must have a gauge fixing $\nabla_\nu A^\nu = 0$. In other words, the massless property of a real photon described by the rotational part of A^μ is directly related to the gauge invariance. A peculiar feature of (23) is that it is valid even for lightlike U^σ with a fixed "gauge" of $\nabla_\nu U^\nu = 0$, which is assured by the orthogonality condition of (11).

A polarization vector represents an important aspect of electromagnetic waves, since electric and magnetic vectors $\vec{E}, \vec{H}$ are a couple of elemental physical factors, at least in the classical electromagnetic theory, in terms of which other physical quantities are expressed. In Fig. 4, we illustrate a plane-polarized electromagnetic wave whose $\vec{E}$ and $\vec{H}$ are parallel (or anti-parallel) to x^3 and x^2 axes, respectively. Then we can readily construct the corresponding wave in the CP model: $\phi = \hat{\phi} sin(kx^0 - kx^1)$ and $\lambda = \hat{\lambda} cos(kx^0 - kx^1 - lx^3)$ give a wave with such S_{03} and S_{31} denoted in Fig. 4. In Fig. 4, E_3 as well as H_2 change in a space-time domain according as $sin(kx^0 - kx^1)$ whose space-time dependence are exactly the same as those of C_0 and C_1 derived from $\phi = \hat{\phi} sin(kx^0 - kx^1)$.

So, we may regard C_0 and C_1 as the polarization vector of plane-polarized "light" wave described by CP vortex dynamics. If we accept this interpretation, then we can say that the polarization vectors of conventional electromagnetic field perpendicular to the associated Poynting vector and those of a dual "electromagnetic" field described by CP vortex dynamics are complementary in the sense that they provide a complete orthogonal basis of 4-dimensional spacetime. It is also interesting to note that this complementarity is directly related to the classical wave-particle duality picture manifested in the energy momentum representation (17). Concerning the longitudinal modes in electromagnetic field, there have been many discussions: within the context of the classical Maxwellian theory, for instance, Lax et al. [59] discussed it in relation to paraxial optics and Cicchitelli et al. [60] further discussed their results and showed that the existence of longitudinal modes can be experimentally proven. Note also that the longitudinally polarized photons as virtual ones, similar to (C^0, C^1) mentioned above, are necessary physical ingredients in the perturbative calculations of quantum electromagnetic interaction theory [61] and one of the authors [62] discussed the physical relevance of the longitudinal mode in the formulation of manifestly covariant quantization of electromagnetic field.

4.4. Further remarks on the longitudinal modes

For the sake of a unified understanding of the longitudinal modes of electromagnetic field, we need clearly to treat both the relativistic and non-relativistic situations on the same footing. In the standard approach to this problem, however, the theoretical frameworks prepared are not along this way of thinking! For instance, in the so-called covariant quantization of gauge fields, it is common to focus upon the purely relativistic situation neglecting the presence of non-relativistic one, as a result what remains as physical modes in the physical Hilbert space are only the transverse modes with positive metric, with the longitudinal modes being confined as unphysical modes. To understand the mutual relations between such physical and unphysical modes, we need to unify both the relativistic microscopic level with high energies and the non-relativistic macroscopic level with low energies. Adopting such a wider formulation, we can observe that the longitudinal modes with negative metric hiding themselves from the high energy regions can show up themselves without the visible effects of negative metrics condensed into classical modes.

4.5. Extension to non-lightlike cases

The arguments developed so far are restricted to the dual vortex structure of the lightlike electromagnetic field whose energy-momentum tensor is given by (16) and that of the associated $S_{\mu\nu}$ in (17). In this section, we show that the newly introduced CP vortex formulation can be extended to the cases where U_μ is spacelike. Timelike U_μ is excluded because of (23). In terms of the vector symbols given in (7), the reduced CP flow vector originally defined in (4) can now be redefined as

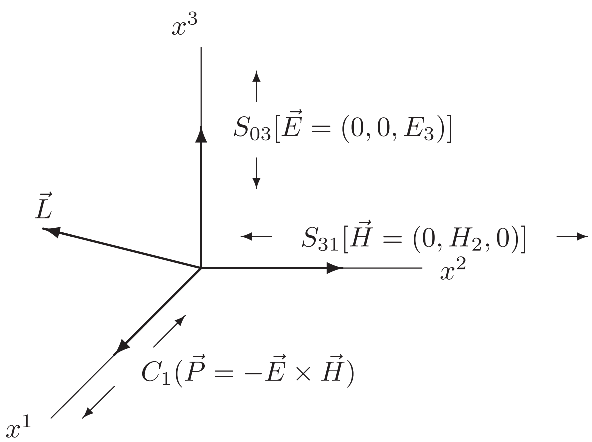

Fig. 4. Dual configuration of $[\vec{E}, \vec{H}, \vec{P}]$ and $[S_{03}, S_{31}, \vec{C}]$.

$$U_\mu = \frac{1}{2}\left(\lambda C_\mu - \phi L_\mu\right). \tag{25}$$

For electromagnetic field, we assume that U_μ satisfies the geodesic equation (1) which is rewritten as

$$S_{\mu\nu}U^\nu + \nabla_\mu V = 0, \tag{26}$$

where $V \equiv U^\nu U_\nu/2$ and $S_{\mu\nu}$ remains to be the same as that given in (7). A natural extension from a lightlike to spacelike CP vortex formalism would be attained through replacing the d'Alembert equation (5) for ϕ by the Klein-Gordon Eq. (6) for λ, that is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\psi - \kappa_0^2\psi = 0. \tag{27}$$

Namely, ϕ and λ satisfy the same equation. In the preceeding discussions on lightlike cases, we confine ourselves to real variables, but, in what follows, for mathematical simplicity let us consider a complex plane wave solution to (27) with the form of $\psi = \exp i(k_\sigma x^\sigma)$ which satisfies the following a couple of equations:

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\psi = -k^\sigma k_\sigma\psi; g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi = -k^\sigma k_\sigma\psi^2, \tag{28}$$

where $\kappa_0^2 = -k^\sigma k_\sigma$. Here, the second equation in (28) should not be confused with that in (6) since we now employ complex representations. Since ϕ and λ satisfy (28), using (7), we have

$$\nabla_\nu C^\nu - \kappa_0^2\phi = 0; C^\nu C_\nu - \kappa_0^2\phi^2 = 0, \tag{29}$$

$$\nabla_\nu L^\nu - \kappa_0^2\lambda = 0; L^\nu L_\nu - \kappa_0^2\lambda^2 = 0. \tag{30}$$

For the directional constraint between $\nabla_\mu\phi$ and $\nabla_\mu\lambda$, we can employ exactly the same conditions as (8) and (11). Note that using (8), (11) and the integrability conditions of $\nabla_\nu L_\mu = \nabla_\mu L_\nu = \nabla_\mu\nabla_\nu\lambda$ and the same for ϕ, we obtain

$$C^\nu\nabla_\nu L_\mu = 0; C^\nu L_\nu = 0, \Rightarrow L^\nu\nabla_\nu C_\mu = 0. \tag{31}$$

With this orthogonality condition, it can readily be shown that U^ν is a divergence free vector, namely, $\nabla_\nu U^\nu = 0$, which can be regarded as the Lorentz gauge in CP vortex formulation. In the use of the second equations in (29, 30) and of the orthogonality condition $C^\nu L_\nu = 0$ in (31), the quantity $V = U^\nu U_\nu/2$ is reduced to

$$V = \left(\frac{1}{2}\right)^3 (\lambda C^\nu - \phi L^\nu)(\lambda C_\nu - \phi L_\nu) = \left(\frac{1}{4}\right)(\kappa_0)^2(\lambda\phi)^2. \tag{32}$$

Now, going back to (26), direct calculations of $S_{\mu\nu}U^\nu$ and $\nabla_\mu V$ yield

$$S_{\mu\nu}U^\nu + \nabla_\mu V = \frac{1}{4}(\lambda\phi)^2\nabla_\mu(\kappa_0)^2 = 0, \tag{33}$$

since κ_0 is not a variable but a constant. By similar simple calculations, we also get

$$U^\sigma\nabla_\sigma(\lambda\phi) = 0; \Omega \equiv S_{\mu\nu}S^{\mu\nu} = 2(\kappa_0)^4(\lambda\phi)^2, \tag{34}$$

from which we obtain an important advection equation:

$$U^\sigma\nabla_\sigma\Omega = 0. \tag{35}$$

In lightlike case, we have looked into the form of energy-momentum tensor given by (17) based on (16). For non-lightlike case, if we follow the conventional electromagnetic knowledge again, it is natural to start with the form:

$$\widehat{T}_\mu^{\ \nu} = -S_{\mu\sigma}S^{\nu\sigma} + \frac{1}{4}S_{\alpha\beta}S^{\alpha\beta}g_\mu^\nu. \tag{36}$$

Through the well-known manipulation in electromagnetic theory, we get

$$\nabla_\nu\widehat{T}_\mu^{\ \nu} = -S_{\mu\sigma}\nabla_\nu S^{\nu\sigma}. \tag{37}$$

By similar manipulations leading to (22), we have

$$\nabla_\nu S^{\nu\sigma} = [-C^\sigma(\nabla_\nu L^\nu) + L^\sigma(\nabla_\nu C^\nu)] + [C^\nu\nabla_\nu L^\sigma - L^\nu\nabla_\nu C^\sigma]. \tag{38}$$

We see that, by using (31), (38) reduces to a form similar to (22) and hence (37) becomes

$$\begin{aligned}\nabla_\nu \widehat{T}_\mu^\nu &= -S_{\mu\sigma}[-C^\sigma(\nabla_\nu L^\nu) + L^\sigma(\nabla_\nu C^\nu)] \\ &= \kappa_0^2 S_{\mu\sigma}(\lambda C^\sigma - \phi L^\sigma) = 2\kappa_0^2 S_{\mu\sigma} U^\sigma.\end{aligned} \tag{39}$$

By using (26), (32) and the second equation in (34), the above equation leads to

$$\nabla_\nu \widehat{T}_\mu^\nu = \nabla_\nu \left(-\frac{1}{4}\Omega g_\mu^\nu\right). \tag{40}$$

Combining (36) and (40), and introducing a new notation: $\widehat{S}_{\mu\nu\sigma\rho} = S_{\mu\nu}S_{\sigma\rho}$, we finally obtain

$$\nabla_\nu \widehat{G}_\mu^\nu = 0; \widehat{G}_\mu^\nu \equiv -\widehat{S}_{\mu\sigma}^{\nu\sigma} + \frac{1}{2}\widehat{S}_{\alpha\beta}^{\alpha\beta} g_\mu^\nu, \tag{41}$$

which is isomorphic to the following Einstein tensor G_μ^ν:

$$\nabla_\nu G_\mu^\nu = 0; G_\mu^\nu \equiv -R_{\mu\sigma}^{\nu\sigma} + \frac{1}{2}R_{\alpha\beta}^{\alpha\beta} g_\mu^\nu = T_\mu^\nu, \tag{42}$$

where $R_{\mu\nu\sigma\rho}$ denotes Riemann tensor and T_μ^ν is an energy-momentum tensor of non-gravitational origin. Since (41) is isomorphic to (42), tensor $\widehat{G}_\mu^\nu$ is well qualified for being an energy-momentum tensor of a new type of electromagnetic field. Recall that "space" and "time" as a couple of conceptual quantities in Newtonian physics were merged into a single notion of flat Minkowski spacetime by the theory of special relativity in which light plays a crucial role. CP vortex model for the new type of electromagnetic field has shown that not only flat but also curved spacetime structure can be regarded as an emergent feature of electromagnetic field with a "mass" term of $S_{\mu\nu}S^{\mu\nu}$. In order to check whether "mass" term Ω is positive or not, we first check the magnitude of V denoted by $\|V\|$ based on (32). Since U^μ is spacelike, $\|V\|$ must be negative, which can be verified by

$$\|V\| = \frac{1}{2^3}(\lambda C_\nu - \phi L_\nu)^*(\lambda C^\nu - \phi L^\nu) = -\frac{1}{4}(\kappa_0)^2(\lambda^*\lambda)(\phi^*\phi), \tag{43}$$

where superscript * denotes the complex conjugate. Then, by combining (43) with the second equation in (34), we have

$$||\Omega|| = 8(\kappa_0)^2||V||, \tag{44}$$

from which we see that $\|\Omega\|$ is also negative.

In order to see the implication of (44), let us introduce a couple of constant dimensional and variable non-dimensional parameters $\widehat{V}_0$ and n with which the squared magnitude of U^μ given at one line below (26), namely, $V = U^\nu U_\nu/2$, is scaled as

$$V = \frac{1}{2}\left(\widehat{V}_0 n\right)^2 v^\nu v_\nu, \tag{45}$$

where v^ν denotes non-dimensional four velocity vector satisfying a spacelike condition of $\|v^\nu v_\nu\| = -1$. Since $-\|\Omega\|$ is a squared magnitude of the vorticity tensor in CP model, we can introduce ω which represents the magnitude of vorticity as $\omega^2 \equiv -\|\Omega\|$. Then, with this ω and (45), Eq. (44) is rewritten as

$$|\tilde{\omega}| = 2(\kappa_0)^2 n, \tag{46}$$

where $\tilde{\omega} \equiv \kappa_0\omega/\widehat{V}_0$. Note that Eq. (46) is isomorphic to the linear relation known as Regge trajectories for resonance particles in the field of high energy particle physics. Actually, since the dimension of the r.h.s. of (46) is l^{-2} which is equal to M^2 in a natural units system, (46) says that the rotational measure defined as $\tilde{\omega}$ is proportional to the magnitude of squared "mass"field. In the case of Regge trajectories, the l.h.s. corresponds to the angular momentum of a resonance particle practice.

4.6. Physical interpretation of CP vortex as a model for off-shell photons

According to our discussion so far, there exist a couple of electromagnetic fields characterized respectively by (23) and (24), which may be called spacelike and timelike electromagnetic fields when we exclude lightlike modes in respective fields. Although both fields satisfy formally the same Lorentz gauge condition of the form: $\nabla_\nu A^\nu = 0$ and $\nabla_\nu U^\nu = 0$, their physical meanings are quite different. While the physical origin of $C_\mu = \nabla_\mu\phi$ in a lightlike mode is discussed in subsections 4.2 and 4.3, the fate of $L_\mu = \nabla_\mu\lambda$ has not been settled yet. Let us consider first the physical meaning of $S_{\mu\nu}$ expressed in a bivector representation in terms of C_μ and L_ν given in (7). Since C^μ can be regarded as the linear momentum vector field of a given current of "photons" as the classical particles, the vorticity tensor $S_{\mu\nu}$ may be formally regarded as the associated angular momentum tensor field if L^ν plays the role of the position vector field of the current, which is consistent with the fact that L^ν must be spacelike in CP vortex formulation. Note also that the physical dimension of U^μ becomes exactly the same as that of A^μ, if $\phi = \nabla_\nu A^\nu$ and the physical dimension of L_μ is that of length! Clearly, in this reinterpretation, $S_{\mu\nu}$ becomes a so-called orbital angular momentum field. One of the striking features of photon missing in the classical electromagnetic theory is its intrinsic spin which is related to circularly polarized (right or left) classical electromagnetic field. So, in this sense, we may say that circularly polarized state is more fundamental than plane-polarized one. Notice that the circular motions of electric and magnetic vectors $\overrightarrow{E}$ and $\overrightarrow{M}$ in such a circularly polarized state naturally induce a current σ^μ whose mathematical expression is quite similar to that of the Poynting vector defined as the exterior product of those two vectors. In subsection 4.1, we referred to quantum-classical correspondence as an important guiding principle bridging the gap between the classical and quantum descriptions. From the viewpoint of quantum-classical correspondence, the introduction of a hypothetical σ^μ current as an intrinsic helicity vector field corresponding to the spin of photons seems to be a plausible component which smoothly connects the classical electromagnetic theory to quantum one and, from this viewpoint, the vector field $U_\mu = \lambda \nabla_\mu \phi$ given in (4) considered to be the vector potential for $S_{\mu\nu}$ can be physically interpreted as the above σ^μ current.

As we have shown in subsections 4.2 and 4.3, the difference between lightlike modes for (23) and (24) is the configuration of polarization vectors and the above argument on identifying circularly polarized state as the classical "photon" accompanying σ^μ current implies that such a state is a combined mode of (23) and (24) and is complete in the sense that its four polarization vectors provide orthogonal basis of the spacetime. When U^μ in (23) becomes spacelike, unlike the case of polaritons corresponding to timelike A^μ in (24), an excited state, when it interacts with a localized disturbance with timelike components in the Fourier-transformed space cannot remain to be a stable on-shell mode [63] even if it should elude the limitation arising from the uncertainty principle $\Delta E \Delta t > \hbar$. However, there exists an important exceptional case to this situation. Since any material field in a steady state is represented by spacelike vectors, the interactions with such fields does not lead to the instability of U^μ field. We think that the manifestation of DP around spatially fixed substances occurs as a result of such a stable interaction with steady state material fields and spacelike U^μ field excited at the characteristic length scale of $(\kappa_0)^{-1}$. A spherically symmetric solution to the Klein-Gordon equation (27) without a temporal differential term is known to yield Yukawa potential whose magnitude decreases exponentially in the radial direction. So far no theoretical attempt including ones referred to in section 2 can successfully reproduce such an experimentally verified damping feature of DP field in near vacuum environment. Thus, CP vortex model proposed in this article seems to be a promising approach to the undiscovered realm of DP.

5. Discussion and summary

For clarifying the essential line of main contents, it would be useful to make the following comments on the energy(-momentum) spectra of field operators and of state vector spaces. As mentioned in Introduction, there is a sharp difference in energy-momentum spectra of field operators between free and interacting fields, which cover a mass hyperboloid $p^2 = m^2$ to characterize the on-shell particles in the former case and the whole p-space in the latter. While the latter situation has been always neglected in the standard discussions in physics, it plays the crucial roles in describing all the interaction processes and also in the consistent formulation of DPs and of DPPs, which has been implemented in Section 4 by the help of $S_{\mu\nu}$ as the Clebsch dual of $F_{\mu\nu}$. Namely, the Clebsch duality relation between $F_{\mu\nu}$ and $S_{\mu\nu}$ can be seen from the viewpoint of duality between visible and invisible aspects, originating from the duality between positive and negative sides of the lightcone $p^2 = 0$.

The next remark to be given is the difference in the energy-momentum spectrum between field operators and state vector spaces, which is important though seldom noted nor mentioned explicitly: the presence of positive/negative energy spectra in field operators should easily be seen by the existence of creation and annihilation operators, which could not be distinguished without the sign difference of energies carried by these operators. In the state vector space, however, such a familiar condition on the vacuum vector $|0\rangle$ as $a|0\rangle = 0$ to characterize the Fock space structure is working as the selection of one-sided spectrum of energy at the level of state vector space. From this situation, such a blind belief seems to start that the energy spectra in quantum theory must always be positive, in combination with which the vacuum vector $|0\rangle$ generating all the state vectors (due to the cyclicity assumption) becomes such a mysterious object as creating everything in spite of its emptiness!! If we move from the vacuum situation to thermal ones, then the right/left symmetry of state vector space of Gibbs state or the modular inversion symmetry valid in the Tomita-Takesaki extension [64] of thermal equilibrium to infinite systems implies that energy spectrum in this situation is symmetric under the sign change of the energy. Actually, the positive energy spectrum is seen to be an exceptional condition to characterize the concept of vacua and it can be easily violated in almost all the physical situations. The scenario explained briefly in Introduction can naturally be understood, once the above two kinds of remarks are accepted: For instance, if you stick to the standard familiar concept of the usual (i.e., massless on-shell) photons, then the concept of off-shell photons or DPs may sound quite strange and unfamiliar. If you start to realize that any kinds of particles will experience their off-shell forms during the mutual interactions, then the existence of off-shell photons or DPs is seen to be quite natural in spite of their being invisible. As for the invisibility, you cannot observe any meaningful interaction processes causing changes on any seemingly "unchangeable" objects, without which this world become frozen!!

Therefore, it may be useful to add a comment on the parallelism of the above ubiquitous changes (of seemingly unchangeable objects via the duality between visible and invisible) with the essence of thermodynamics involving stability and instability! When we see any processes of working thermal engines, *e.g.*, Carnot cycles for instance, it is one of the universal features that the cycle contains the subprocesses in which heat and/or energy is absorbed and emitted. If we take the starting point of exerting the work as the origin of the energy levels, then the absorbing process is seen to correspond to the states with negative energy spectrum and the working one with energy emitted to those with positive energy spectrum. From this viewpoint, the usual formulation of the vacuum states in relativistic QFT can be seen to be quite unrealistic, where the existence of states with negative energy spectrum is denied and where the state with the zero energy is identified with the vacuum state. In this context, the usual $F_{\mu\nu}$ can be seen to correspond to the processes emitting heat to the cooler reservoirs exerting the work onto the external world, and $S_{\mu\nu}$ to those absorbing heat from the hotter heat reservoirs. Similarly in the example of Mandelstam variables for scattering processes, the *s*-channel with incoming and outgoing particles can be put in analogy with the processes with energy emitted and *t*-channel with energy-momentum transfer caused by interaction potentials with those absorbing energy. If the former aspect is also put in correspondence with the branches of back and forth fluctuations in the steepest descent method, then the latter one with falling down slope, along which energy absorbing process let the working system climb up to the saddle point. These two kinds of examples can be unified actually into the context of large deviation principle [65,66], from the viewpoint of which the essence of DP will be well understood.

Acknowledgements

The authors' work presented here were partly supported by Research Origin of Dressed Photons (RODreP), the JSPS Core-to-Core Program (A. Advanced Research Networks) and NICHIA Corporation.

References

[1] R.F. Streater, A.S. Wightman, PCT, Spin and Statistics, and All That, Princeton University Press, 2000.
[2] I. Ojima, H. Saigo, Mathematics 3 (2015) 897.
[3] M. Ohtsu, Opt. Rev. 21 (2014) 905.
[4] D. Pines, Elementary Excitation in Solids, Perseus Books, Reading, 1999.
[5] M. Ohtsu, K. Kobayashi, Optical Near Fields, Springer, Heidelberg, 2004.
[6] M. Thomson, Modern Particle Physics, Cambridge Univ. Press, Cambridge, 2013, pp. 117–119.
[7] R.P. Feynman, The Theory of Fundamental Processes, W. A. Benjamin, New York, 1962, pp. 95–100.
[8] M. Ohtsu, H. Hori, Near-Field Nano-Optics, Kluwer Academic/Plenum Publishers, New York, 1999, pp. 29–31.
[9] K. Kobayashi, Surf. Sci. 30 (2009) 638.
[10] M. Ohtsu, Dressed Photons, Springer, Heidelberg, 2013, pp. 11–36.
[11] Y. Tanaka, Phys. E 40 (2007) 297.
[12] H. Hyuga, H. Ohtsubo, Nucl. Phys. A 294 (1978) 348.
[13] K. Kobayashi, M. Ohtsu, J. Microsc. 19 (1999) 2494.
[14] S. Sangu, K. Kobayashi, M. Ohtsu, J. Microsc. 202 (2001) 279.
[15] S. Sangu, K. Kobayashi, S. Shojiguchi, M. Ohtsu, Phys. Rev. B 69 (2004) 1153343.
[16] T. Kawazoe, K. Kobayashi, S. Sangu, M. Ohtsu, Appl. Phys. Lett. 82 (2003) 2957.
[17] W. Nomura, T. Yatsui, T. Kawazoe, M. Ohtsu, J. Nanophot. 1 (2007) 011591.
[18] T. Kawazoe, K. Kobayashi, M. Ohtsu, Appl. Phys. Lett. 86 (2005) 103102.
[19] T. Kawazoe, M. Ohtsu, S. Aso, Y. Sawado, Y. Hosoda, K. Yoshizawa, K. Akahane, N. Yamamoto, M. Naruse, Appl. Phys. B 103 (2011) 537.
[20] T. Yatsui, K. Hirata, W. Nomura, Y. Tabata, M. Ohtsu, Appl. Phys. B 93 (2008) 55.
[21] T. Yatsui, K. Hirata, Y. Tabata, W. Nomura, T. Kawazoe, M. Naruse, M. Ohtsu, Nanotechnol 21 (2001) 355303.
[22] W. Nomura, T. Yatsui, T. Kawazoe, N. Tate, M. Ohtsu, Appl. Phys. A 121 (2015) 1403.
[23] R. Teki, A.J. Kadaksham, M. House, J. Harris-Jones, A. Ma, S.V. Babu, A. Hariprasad, P. Dumas, R. Jenkins, J. Provine, A. Richmann, J. Stowers, S. Meyers, U. Dietze, T. Kusumoto, T. Yatsui, M. Ohtsu, in: Proc. SPIE 8322, 2012, p. 83220B.
[24] T. Yatsui, K. Hirata, Y. Tabata, Y. Miyake, Y. Akita, M. Yoshimoto, W. Nomura, T. Kawazoe, M. Naruse, M. Ohtsu, Appl. Phys. B 103 (2011) 527.
[25] T. Yatsui, W. Nomura, M. Ohtsu, Adv. Opt. Technol. 2015 (2015). Article ID 701802.
[26] T. Yatsui, W. Nomura, F. Stehlin, O. Soppera, M. Naruse, M. Ohtsu, Beilstein J. Nanotechnol. 4 (2013) 875.
[27] T. Yatsui, W. Nomura, M. Naruse, M. Ohtsu, J. Phys. D. 45 (2012) 475302.
[28] T. Kawazoe, H. Fujiwara, K. Kobayashi, M. Ohtsu, IEEE J. Select. Top. Quant. Electron 15 (2009) 1380.
[29] H. Fujiwara, T. Kawazoe, M. Ohtsu, Appl. Phys. B 98 (2010) 283.
[30] S. Yukutake, T. Kawazoe, T. Yatsui, W. Nomura, K. Kitamura, M. Ohtsu, Appl. Phys. B 99 (2010) 415.
[31] H. Tanaka, T. Kawazoe, M. Ohtsu, Appl. Phys. B 108 (2012) 51.
[32] T. Kawazoe, M.A. Mueed, M. Ohtsu, Appl. Phys. B 104 (2011) 747.
[33] M.A. Tran, T. Kawazoe, M. Ohtsu, Appl. Phys. A 115 (2014) 105.
[34] M. Yanaguchi, T. Kawazoe, T. Yatsui, M. Ohtsu, Appl. Phys. A 121 (2015) 1389.
[35] M. Ohtsu, Silicon Light-emitting Diodes and Lasers, Springer, Heidelberg, 2016, pp. 15–42.
[36] J.H. Kim, T. Kawazoe, M. Ohtsu, Adv. Opt. Technol. 2015 (2015). Article ID 236014.
[37] J.H. Kim, T. Kawazoe, M. Ohtsu, Appl. Phys. A 121 (2015) 1395.
[38] T. Kawazoe, M. Ohtsu, Appl. Phys. A 15 (2014) 127.
[39] Q.H. Vo, T. Kawazoe, M. Ohtsu, in: Extended Abstracts of the 61st Spring Meeting on Applied Physics and Related Societies, 2014, 18A–F12–10.
[40] T. Kawazoe, M. Ohtsu, in: Extended Abstracts of the 59st Spring Meeting on Applied Physics and Related Societies, 2012, 17p–B11–1.
[41] N. Wada, T. Kawazoe, M. Ohtsu, Appl. Phys. B 108 (2012) 25.
[42] T. Kawazoe, M. Ohtsu, K. Akahane, N. Yamamoto, Appl. Phys. B 107 (2012) 659.
[43] H. Tanaka, T. Kawazoe, M. Ohtsu, K. Akahane, Fluoresc. Mat. 1 (2015) 1.
[44] H. Tanaka, T. Kawazoe, M. Ohtsu, K. Akahane, N. Yamamoto, Appl. Phys. A 121 (2015) 1377.
[45] K. Kitamura, T. Kawazoe, M. Ohtsu, Appl. Phys. B 107 (2012) 293.
[46] N. Tate, T. Kawazoe, W. Nomura, M. Ohtsu, Sci. Rep. 5 (2015) 12762.
[47] N. Tate, T. Kawazoe, S. Nakashima, W. Nomura, M. Ohtsu, in: Proceedings the 22nd International Display Workshops (IDW15), 2015, p. 1066.
[48] T. Kawazoe, N. Tate, M. Ohtsu, in: Proceedings of the 22nd International Display Workshops (IDW15), 2015, p. 1081.
[49] H. Tanaka, T. Kawazoe, M. Ohtsu, in: Extended Abstracts of the 63rd Spring Meeting on Applied Physics and Related Societies, 2016, 19a–S622–8.
[50] T. Kawazoe, K. Hashimoto, S. Sugiura, in: Abstract of the EMN Nanocrystals Meeting, 2016, pp. 9–11.
[51] T. Kawazoe, K. Nishioka, M. Ohtsu, Appl. Phys. A 121 (2015) 1409.
[52] M. Ohtsu, T. Kawazoe, Silicon light emitting diodes and lasers using dressed photons, in: M. Ohtsu, T. Yatsui (Eds.), Progress in Nanophotonics, vol. 3, Springer, Heidelberg, 2015, pp. 1–56.
[53] I. Ojima, in: Proceedings of the International Conference of Stochastic Analysis, 2005. arXiv:math-ph/0502038.
[54] S. MacLane, Categories for the Working Mathematician, Springer, Heidelberg, 1971.
[55] I. Ojima, Micro-macro duality in quantum physics, in: T. Hida (Ed.), Stochastic Analysis: Classical and Quantum -Perspectives of White Noise Theory, World Scientific, 2005, pp. 143–161.
[56] S.H. Lamb, Hydrodynamics, sixth ed., Cambridge University Press, 1932.
[57] J. Marsden, A. Weinstein, Physica 7D (1983) 305–323.

[58] H. Sakuma, Note on a Novel Vortex Dynamics of Spacetime as a Heuristic Model of the Vacuum Energy, 2014 arXiv:1409.2607[gr-qc].
[59] M. Lax, W.H. Louisell, W.B. McKnight, Phys. Rev. A 11 (1975) 1365–1370.
[60] L. Cicchitelli, H. Hora, R. Postle, Phys. Rev. A 41 (1990) 3727–3732.
[61] I.J.R. Aitchison, A.J.G. Hey, Gauge Theories in Particle Physics, second ed., Adam Hilger, Bristol and Philadelphia, 1989.
[62] I. Ojima, Nakanishi-lautrup B field, crossed product and duality, in: RIMS Workshop: Research on Quantum Field Theory, 2006, pp. 29–37.
[63] Y. Aharonov, A. Komar, L. Susskind, Phys. Rev. 182 (1969) 1400–1403.
[64] O. Bratteli, D. Robinson, Operator Algebras and Statistical Mechanics, second ed., vol. 1, Springer-Verlag, 1987.
[65] R. Ellis, Entropy, Large Deviations, and Statistical Mechanics, Springer-Verlag, 1985.
[66] J. Deuschel, D. Stroock, Large Deviations, Academic Press, 1989.