# Embarking on theoretical studies for off-shell science guided by dressed photons

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## Abstract

By noting that the dressed photon (DP) is a quantum field whose energy–momentum relation deviates from the mass-shell, novel theoretical studies of so-called off-shell science have been launched. This article reviews recent progress in these studies. After reviewing the characteristics of the DP as an off-shell quantum field, theories having a physical basis are introduced. These theories are an electromagnetic response theory and a theory based on spatio-temporal vortex hydrodynamics. Next, theories having a mathematical basis are introduced, and these can serve as helpful tools for gaining a deep understanding of the concepts of the physics-based theories above. These theories are a quantum probability theory and a quantum walk model. As a further helpful tool, a quantum measurement theory is introduced. A theory based on micro–macro duality is demonstrated, which serves as the foundation to embark on a stuy of off-shell science. Correlations among the theories reviewed here are also shown.

## 1 Introduction

Studies on dressed photons (DPs) have found that the DP is a quantum field created by light–matter interaction in a nanometric space [1]. Some of its unique characteristics, outline below, have been demonstrated by experimental studies[*]:

[a] The DP is a field composed of photons and electrons (or excitons). It is created and localized at the boundary or at the singular point of a nanometric material, i.e., on the material surface or at an impurity atom in the material.

[b] The energy and momentum of the DP range widely.

[c] The DP is a quantum field off the mass-shell (“off-shell quantum field” for short).

[d] Electrons and excitons can be excited and de-excited by the DP even under non-resonant condition.

[e] The DP energy is exchanged and transferred between nano-materials when they are

located in the close proximity to each other.
[f] The DP field is disturbed when it is measured by inserting a probe into the field.
[g] The DP is transferred in an autonomous manner between the nano-materials.
[h] The spatial distribution of DPs on a material surface has a hierarchical structure.

Characteristics [a]–[h] above have been applied to the invention of novel optical logic gate devices, nano-fabrication technology, and energy conversion technology [2]. They have also been applied to the invention of novel high-power lasers and light-emitting diodes using crystalline Si, even though Si is an indirect-transition-type semiconductor [3]. These applications demonstrate the advent of a revolutionary generic technology that could never have been realized as long as conventional light (free photons) is used (Fig.1) [4].

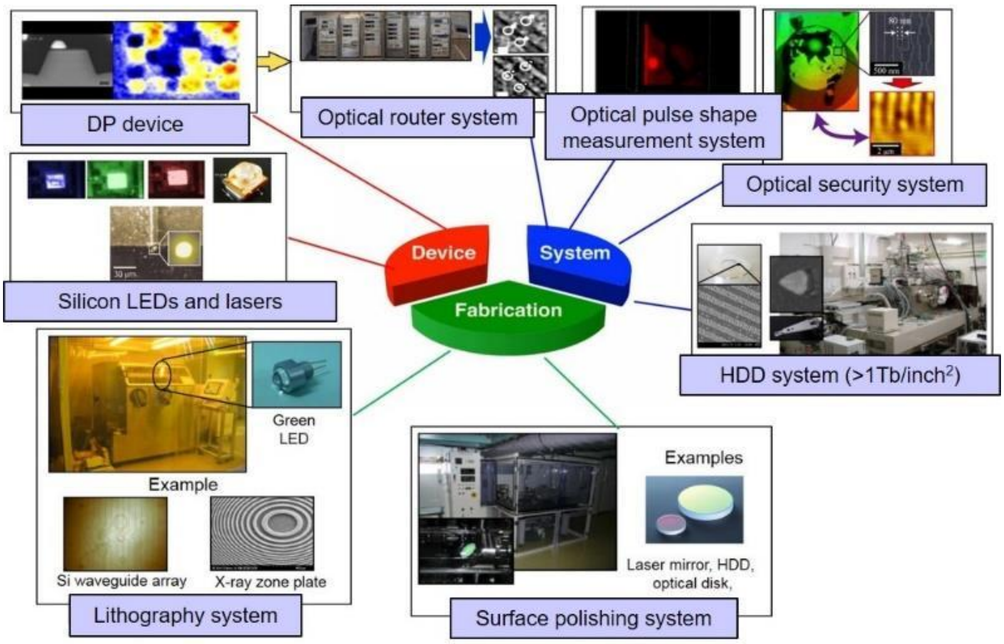

Fig.1 Generic technology that has emerged by applying the unique characteristics of the DP [4].

The advent of this technology suggests that the study of the DP has entered a new era in which the construction of advanced theories will be indispensable for accelerating technological progress.

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(*) Characteristic [a] can be considered as the origin of characteristics [b] and [e]. Furthermore, characteristic [b] can be considered as the origin of characteristics [c] and [d], and characteristic [e] that of characteristic [f].

## 2 The dressed photon as an off-shell quantum field

Characteristics [a]–[h] in Section 1 cannot be described by conventional optical theories. This is because these theories have treated only a photon in vacuum (free photon) and in a macroscopic material, whose dispersion relation is on the mass-shell (“on-shell”, for short). It has been popularly known that massless particles with non-zero spin, such as free photons, cannot be localized in space, in the sense that the position operator cannot be well-defined [5, 6]. However, it turns out to be natural to consider localized photons when the effective mass of photons, created by the light–matter interactions, is taken into account. Especially in the case of nano-materials, space–time localization and energy–momentum fluctuation provide brand new aspects of light. A photon in such a context is called a DP [1].

For a theoretical definition of the DP, the “off-shell” nature of the interaction has to be considered. That is, the DP is an off-shell quantum field that conspicuously deviates from the mass-shell in the dispersion relation (Fig. 2). As has been well known, quantum field theories cannot be formulated without off-shell entities. In other words, the traditional particle description has failed to treat the composite system of quantum fields. Hence, DPs must be entities that are very different from Einstein’s quanta of light, or free photons.

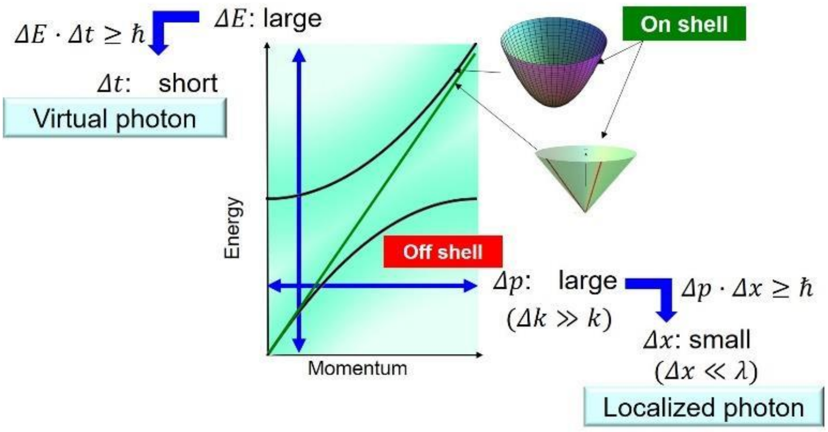

Fig.2 The dispersion relation of the electromagnetic field on-shell or off-shell. $E$, $t$, $p$, $x$, $k$, and $\lambda$ are energy, time, momentum, position, wave-number, and wavelength, respectively. $\hbar$ is Planck ‘s constant $h$ divided by $2\pi$.

Here, a fundamental question arises: How can the DP be described as an individual entity? As long as one sticks to the notion of individual entities as irreducible on-shell particles, it is impossible to treat the DP as an individual entity. However, a more general perspective, advocated by Ojima [7], has shown that macroscopic physical phenomena can emerge out of a condensation of microscopic off-shell entities.

By following this perspective, a basic idea can be proposed: In the interaction between light and a nano-material, certain families of modes of the composite system will behave as individuals. This behavior suggests that the DP is the quantum field of a composite system in which an electromagnetic field and an electron (or an exciton) interact in a nanometric space. Furthermore, it is a virtual field localized in a nanometric space within a short time duration. Thus, the DP is a quantum field whose nature is incompatible with that of an on-shell photon. This means that conventional optical theories are incapable of giving a systematic description of characteristics [a]–[h] above. Fortunately, however, as will be reviewed in Sections 3 and 4, novel theoretical studies have been commenced in order to draw a precise theoretical picture of the DP to provide a systematic description of these characteristics.

Several hints have been found to construct such novel theories by noting that the virtual photon plays an essential role in the electromagnetic Coulomb interactions. They are:

[A] The longitudinal mode of an electromagnetic field (the longitudinal wave) contributes to the Coulomb interaction [8].

[B] The field interaction accompanies the 4-momentum [9].

[C] The spacelike field is not spatially localized because it behaves as a stable wave. However, it becomes unstable and can localize if it interacts with a timelike field [10].

By referring to these hints, novel theoretical studies relying on physical as well as mathematical bases have commenced [11].

## 3 Theories having a physical basis

This section reviews two examples of novel theories constructed on a physical basis. One is a response theory based on classical electromagnetics. The other is a theory based on spatio-temporal vortex hydrodynamics, supported by relativity theory.

### 3.1 Electromagnetic response theory

A novel response theory was constructed using an electromagnetic response function. As shown by Fig. 3, a nano-particle 1 (NP1) serves as a light source. It corresponds to a fiber probe that creates a DP on its tip. A nano-particle 2 (NP2) is illuminated by the light emitted from NP1. Since NP2 is placed in close proximity to NP1 in the case of

Fig. 3(a) ($d \ll \lambda$: $d$ is the separation between NP1 and NP2, and $\lambda$ is the wavelength of the light), the electron in NP2 responds not only to the transverse electric field ($\boldsymbol{E}^{(trans)}$) of the light but also to its longitudinal electric field ($\boldsymbol{E}^{(lon)}$).

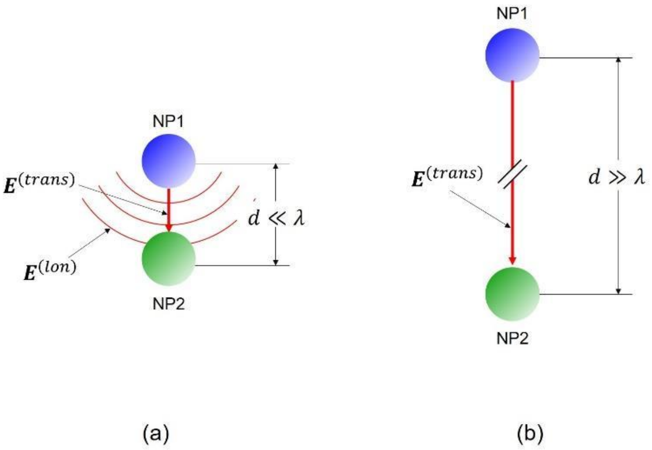

Fig.3 Schematic explanation of the theoretical model.
(a) Near-field condition. (b) Far-field condition.

The field $\boldsymbol{E}^{(trans)}$ is created by a transverse current in NP1. It is a radiative field that follows the Ampère-Maxwell law. It has been popularly known that conventional optical phenomena occur and are observed by this field even under far-field condition ($d \gg \lambda$: Fig.3(b)). On the other hand, $\boldsymbol{E}^{(lon)}$ is a non-radiative field, which is created by an electric charge in NP1 and follows Coulomb's law. It causes unique optical phenomena to occur only under the near-field condition ($d \ll \lambda$: (Fig.3(a)).

The Schrödinger equation was used to describe the electronic state in NP2 under light illumination, for which both the scalar ($\phi$) and vector ($\boldsymbol{A}$) potentials, originated respectively from $\boldsymbol{E}^{(lon)}$ and $\boldsymbol{E}^{(trans)}$, were adopted to represent the light–matter interaction. The potentials $\phi$ and $\boldsymbol{A}$ appeared with the linear and quadratic forms, respectively, in the relevant equations. The difference in their forms originated from the non-relativistic nature of the system under study.

For describing the phenomena that originated from the DP, the present theory treats $\phi$ equivalently with $\boldsymbol{A}$. It should be noted here that the conventional response theories have eliminated $\phi$ by transforming it to the two-body Coulomb interaction potential. Being different from them, the present theory treats not only $\boldsymbol{A}$ but also $\phi$ as the "cause" of the response. For this treatment, a semiclassical response theory was constructed to derive the electric charge density and the electric current density, induced as the "response" of the electron in NP2. For representing the response, the single susceptibility was calculated by a method based on density functional theory.

From the time-integral of the commutator in the expressions for the electric charge and current densities in the NP2 (eqs. (6.17) and (6.18) in ref. [12], respectively), variables representing the energies appeared in the denominators of the derived fractions. These fractions denote the resonance phenomena because they diverge to infinity by tuning the photon energy. However, eq. (6.18) in ref. [12] also has non-resonant fractions that originated from the non-relativistic nature of the system. Since $\phi$ and $\boldsymbol{A}$ are respectively represented by $\boldsymbol{E}^{(lon)}$ and $\boldsymbol{E}^{(trans)}$, the susceptibility can be derived from the proportional constants between the induced electric charge density (current density) and $\boldsymbol{E}^{(lon)}$ ($\boldsymbol{E}^{(trans)}$).

In the case of the electric dipole-allowed transition between the two-energy levels of the electron, the cause of the response can be attributed to the total electric field $\boldsymbol{E}^{(total)}(=\boldsymbol{E}^{(lon)}+\boldsymbol{E}^{(trans)})$ when the system is under the far-field and resonant conditions. That is, the cause can be represented by $\boldsymbol{E}^{(total)}$ and the electric permittivity. However, in the case of phenomena that originated from the DP, especially the one that occurred under the non-resonant condition, it should be noted that $\boldsymbol{E}^{(total)}$ cannot serve as the cause of the response. This means that neither the constitutive equation using electric permittivity and magnetic permeability nor numerical simulation using the finite-domain and time-domain (FDTD) method are valid.

In the case of the electric dipole-forbidden transition, on the other hand, only the non-resonant term contributes. It was confirmed that this term led to an equation that is equivalent to the London constitutive equation for the Meissner effect. Thus, in this case, $\boldsymbol{A}$ serves as the cause of the response.

NP2 was assumed to be a nonmetallic material in the present theoretical study, and this has also been employed in a series of experimental studies on the DP [2,3]. A metallic material was not employed here because it is unsuitable for creating the DP. This is because the temporal coherence of the incident electromagnetic field is lost within a very short time due to the very short transverse relaxation time of an electron in the metal.

The constructed theory successfully described the excitation and de-excitation of electrons or excitons, the contribution of phonons, and the magnetic interactions found in experimental studies of the DP under the non-resonant condition of light–matter interaction. The main derived result is that: $\boldsymbol{E}^{(lon)}$ caused a large electronic response. Furthermore, the non-resonant term of the electric susceptibility was much larger than the resonant term [12], which explains characteristics [d] and [e].

As is understood from the discussions above, $\boldsymbol{E}^{(total)}$ failed to describe the response of NP2 in the case where the conditions of non-resonance and near-field illumination/measurement are simultaneously met. This failure was never found in previous studies on the DP because only $\boldsymbol{A}$ was employed as the cause. The present theory succeeded in specifying that $\phi$ is indispensable for describing the bound state of the electron for which the quantum many-electron effect (i.e., an exchange/correlation interaction) was taken into account.

In future studies, more advanced response theories are expected to systematically explain characteristic [e].

### 3.2 Theory based on spatio-temporal vortex hydrodynamics

Characteristic [b] suggests that the inequality $E < cp$ holds ($E$, $c$, and $p$ are the energy, speed, and momentum of an electromagnetic field, respectively), which means that the field can exist in the spacelike domain of the Minkowski space. In addition, [B] in Section 2 suggests that a timelike-support and spacelike-support of the 4-momenta are required to describe the interacting fields. By referring to these suggestions and also to [C] in Section 2, it can be conjectured that the DP can be created by the interaction between the fields in the timelike and the spacelike domains.

Prompted by this conjecture, a novel theory has been constructed by focusing on the similarity in formulation between vortex hydrodynamics and electromagnetics [13]. For this construction, it was also noted that the contribution of the spacelike momenta was indispensable for the interaction between the quantum fields to occur [9].

Conventional classical theories have claimed that the Coulomb mode played a principal role in the electromagnetic interaction and that the longitudinal wave was a physically existing mode [8, 14-16] (refer also to [A] in Section 2)[*]. In contrast, conventional theories of quantum electrodynamics have excluded the longitudinal wave as a “non-physical mode” even though it had a close relation with the Coulomb mode. Instead, they have introduced the exchange of virtual photons into the theoretical model for describing the electromagnetic interaction. This contrast suggests that a rift exists between the classical and quantum explanations above. This problem should be solved to draw a consistent physical picture of the DP that exists in an intermediate area between the classical and quantum worlds.

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(*) This claim is consistent with the discussions in Section 3.1, where it was pointed out that the longitudinal electric field $\boldsymbol{E}^{(lon)}$ (and also $\phi$) plays an essential role in the phenomena that originated from the DP.

It should be pointed out that the theory of micro-macro duality (Subsection 4.2 (2)) has already explained how to connect the classical and quantum worlds, by which a clue to solve the problem above can be found. The principal advantage of this theory is the capability of analyzing versatile structures of quantum fields with infinite degrees of freedom. This theory has demonstrated that the two worlds above coexist in the sense that the classical–quantum correspondence is mathematically guaranteed. The main purpose of the present subsection is to describe the electromagnetic interaction by adopting the micro–macro duality theory. It is expected that this description can systematically demonstrate the contributions of the longitudinal wave and the spacelike 4-momenta for drawing a physical picture of the DP.

For this demonstration, a novel mathematical expression, called the Clebsch representation, is adopted for the 4-vector potential of the electromagnetic field. The Clebsch representation is a method involving the use of Clebsch variables for representing the velocity vector field $v_{\mu}$ that is introduced to analyze the Hamiltonian of a barotropic fluid. It should be noted here that the mathematical structure (eq. (1a)) of the 4-vector potential $\boldsymbol{A}$ of the skew-symmetric electromagnetic field is similar to that of the equation of motion (eq. 1(b)) for a barotropic fluid based on relativity theory:

$$F_{\mu\nu}\partial^{\nu}\phi = 0, \tag{1a}$$

$$\omega_{\mu\nu}v^{\nu} = 0, \tag{1b}$$

where $F_{\mu\nu}$ denotes the skew-symmetric transverse electromagnetic field, and $\omega_{\mu\nu}$ is the skew-symmetric vorticity defined by the rotation of the velocity field $v^{\nu}$. This similarity is due to the fact that the scalar field $\phi\left(=\partial_{\nu}A^{\nu}\right)$ satisfies the wave equation and its gradient vector $\partial_{\nu}\phi$ is parallel to the propagation direction of the wave (normal to the electric and magnetic fields).

Next, using the two-variable ($\lambda$ and $\phi$) Clebsch representation ($U_{\mu} = \lambda\partial_{\mu}\phi$), the $v^{\nu}$ in eq. (1b) is regarded as the vector potential of the electromagnetic field. Here, $U_{\mu}$ denotes the Clebsch parameterized 4-vector potential that is parallel to the 4-Poynting vector. Since $\omega_{\mu\nu}$ in eq. (1b) can also be regarded as denoting the electromagnetic field, it is represented by the skew-symmetric field

$$S_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu . \tag{2}$$

Furthermore, the following two equations are derived, whose mathematical structure is similar to that of the Maxwell equation:

$$\partial^\nu \partial_\nu \lambda^\mu - \kappa^2 \lambda^\mu = 0 , \tag{3a}$$

$$\partial^\nu \lambda \partial_\nu \phi = 0 . \tag{3b}$$

Here, eqs. 3(a) and (b) indicate that $\lambda$ follows a spatial Klein-Gordon (KG) equation, and that the two vectors ($\partial^\nu \lambda$ and $\partial_\nu \phi$) are normal to each other, respectively. Using the vector $U_\nu$, these equations can be rewritten as

$$\partial^\nu \partial_\nu U^\mu - \kappa^2 U^\mu = 0 . \tag{4}$$

The field, represented by $U_\mu$, can be called the Clebsch dual (CD) field by comparison with $A^\mu$ that satisfies the Proca equation

$$\partial^\nu \partial_\nu A^\mu + \kappa^2 A^\mu = 0 . \tag{5}$$

The energy-momentum tensor $T_\mu^\nu$ for $S^{\mu\nu}$ is expressed as

$$T_\mu^\nu = -S_{\mu\sigma} S^{\nu\sigma} = \rho C_\mu C^\nu , \tag{6}$$

where $\rho \equiv -\partial^\mu \lambda \partial_\mu \lambda$ denotes a spacelike vector, being proportional to the spacelike momentum. $C_\mu \equiv \partial_\mu \phi$ represents a longitudinal wave. The middle part of eq. (6) has the same form as that of the conventional electromagnetic field. The right-hand side is given by the product of $\rho$ and $C_\mu C^\nu$, which shows that the Clebsch representation succeeded in including two essential elements (the spacelike momentum and the longitudinal wave) in the equations.

Although $U_\mu$ was a null vector in the discussion above, it can be extended to the spacelike domain so that $T_{\mu}^{\nu}$ can be represented by

$$T_{\mu}^{\nu} = -S_{\mu\sigma}S^{\nu\sigma} + S_{\alpha\beta}S^{\alpha\beta}g_{\mu}^{\nu}. \tag{7}$$

The mathematical form of the right-hand side is equivalent to the curvature term in the Einstein equation. It should be pointed out that this equivalency was derived by breaking the U(1) gauge symmetry for extending the CD field to the spacelike domain. Equation (7) is acceptable because the CD field plays the role of the basic mode to represent the spacelike 4-momenta of the interacting fields and because the inherent feature of the relativistic field is represented by its space-time structure.

In order to apply the concept of the CD field above to draw the physical picture of the DP, several points should be noted: The spatially homogeneous spacelike momentum field becomes unstable if it interacts with the timelike momentum field, as was shown in [C] of Section 2. By such an interaction, the timelike and spacelike momentum fields can be transformed between each other, and, as a result, the spatial structures of the fields are significantly deformed. Although such a transformation occurs throughout the whole of the interacting area, it occurs more conspicuously at a singular point of the material, such as at the surface of the material or at the impurity atoms in the material (characteristic [a]).

Several discussions were made to describe this transformation: When the timelike momentum vector satisfies the timelike KG equation, its solution takes the form of a homogeneous wave. Such a homogeneous wavy solution can be also derived from the spacelike KG equation satisfied by the spacelike momentum vector. Since the constants in the KG equation represent the physical quantities of the material under study, the transformation between the timelike and spacelike vectors can be expressed by reversing the signs of these constants.

The information derived by these discussions is:

1) The complex-conjugate amplitudes

$$S_{0r}^{\dagger} = \frac{\omega}{c}R'\exp\left[\frac{\omega}{c}x^{0}\right], \quad S_{0r} = -\frac{\omega}{c}R'\exp\left[-\frac{\omega}{c}x^{0}\right] \tag{8}$$

of the derived CD field correspond to the creation ($\hat{a}^{\dagger}$) and annihilation ($\hat{a}$) operators of the quantum harmonic oscillator, respectively. Here, $\omega$ is the angular frequency. $R'$ is the radial component of the solution of the KG equation. This correspondence enabled the definition of the normal mode of the electromagnetic field in a sub-wavelength-sized field, which had been impossible with the previous theory [17].

2) The CD field represents a longitudinal wave (the complex-conjugate amplitudes $C_\mu$ and $C_\mu^*$) that is accompanied by the components ($L_\mu (= \partial_\mu \lambda)$ and $L_\mu^*$) satisfying the KG equation in the spacelike domain (Fig. 4).

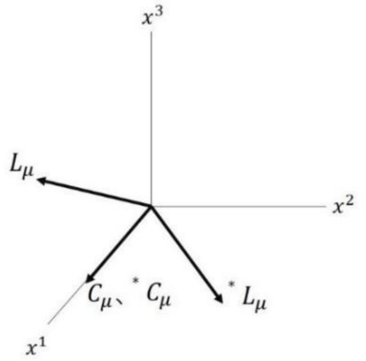

Fig.4 Directions of the amplitudes of the longitudinal wave ( $C_\mu$ and $C_\mu^*$ ) and those of the accompanying components ($L_\mu$ and $L_\mu^*$).

$x^1$ represents the propagation direction of the electromagnetic wave.

3) The components ($L_\mu$ and $L_\mu^*$) become temporally unstable due to the interaction with the field in the timelike domain. As a result, they are created or annihilated within a very short duration, which means that the CD field corresponds to a virtual photon.

4) The transverse wave of the CD field is converted to a longitudinal wave at the material surface. This means that the material surface serves as the source of a longitudinal wave, thus successfully describing characteristic [a].

5) The spatial profile of the field is described by a Yukawa-type function, which can be understood by replacing $x^0$ in $S_{0r}$ of eq.(8) by $x^1$ , $x^2$ , or $x^3$ . As a result, characteristic [a] was also described. This means that the DP is a localized quantum field, created as a result of the transformation of the spacelike momentum field to the timelike field at a singular point of the material.

6) The DP can be represented by the superposition of the longitudinal waves of the CD field. This representation is possible because these waves behave as normal modes. It

should be pointed out that the virtual photon behavior of the components ($L_\mu$ and $L_\mu^*$), accompanying this longitudinal wave, is nothing less than the origin of this successful representation.

These findings 1) – 6) were derived by adopting the longitudinal wave in the present theory as a physical mode. Future progress is expected to explain characteristics [g] and [h], and also to establish the theory of the fully quantum optical version.

## 4 Theories having a mathematical basis

It is expected that mathematics-based theories will serve as invaluable guides for gaining a deep understanding of the concepts of the physics-based theories for the phenomena that originate from the DP. Examples of these theories are the quantum probability theory and the quantum measurement theory, which are reviewed in this section. Also demonstrated is a theory based on micro–macro duality, which serves as a foundation for embarking on theoretical studies of off-shell science.

### 4.1 Quantum probability theory

Quantum probability theory has been constructed by noting characteristic [b] above [18]. This theory focuses on the families of the higher and lower energy–momentum modes for investigating phenomena that cannot be analyzed by conventional on-shell theories. The family of higher modes of the composite system is created as a result of light–matter interaction and behaves like an individual entity. This entity can be defined as the DP. The family of lower modes serves as a kind of heat-bath.

Since no a priori strict boundary between the higher and lower modes exists, it is required to investigate the asymptotic behavior of modes where the energy–momentum becomes large. In other words, the core of a mathematical theory for the DP is nothing but a kind of quantum-classical correspondence for describing an asymptotic state that appears as its quantum number increases to infinity. Hence, some general frameworks are required for both quantum/micro and classical/macro systems. Fortunately, a mathematical theory that meets this requirement has been constructed, that is, the quantum probability theory. The intermediate realm, appearing between the micro- and the macro-systems, has been successfully described by this theory.

As has been popularly known, a quantum harmonic oscillator with a large quantum number behaves very much like a classical harmonic oscillator. The composite system created by light–matter interaction is considered to be a typical example of such a quantum harmonic oscillator. This consideration and the definition of the DP above lead to the fact that the time averaged distribution of the position of the DP can be governed by an arcsine law. Note that each mode of the DP gains an effective mass by the interaction between the light and nano-material, and thus, it is not paradoxical to consider the position of the DP. Moreover, since the size of the nano-material is much less than the wavelength of light, the variance of the distribution will be determined by this size. The stronger the interaction, the higher the energy at a suitable boundary between the family of the higher mode (DP) and that of the lower mode (heat bath). Hence, it is expected that the arcsine law will represent a sufficiently accurate distribution of the DP when the interaction is sufficiently strong.

Since the arcsine function has a twin-peaked profile, the probability of finding the DP will be the highest at the singular point, which is the reason why localization of the DP occurs at the boundary. This localization feature is quite consistent with the experimental results acquired so far [19].

Here, let us take as the most fundamental example the localization of the DP in a fiber probe [20]. The three-dimensional density of the DP can be expressed by an arcsine function

$$f(x)=C\frac{1}{S(x)\sqrt{2-x^2}}, \tag{9}$$

where $C$ and $S(x)$ respectively denote the normalization constant and the cross-sectional area of the fiber probe on which the DP is created. The localization of the DP at the tip of the fiber probe, and furthermore, at the position of the impurity atoms in the material were successfully described based on the twin-peaked spatial feature (peaks at $x=\pm\sqrt{2}$ in eq. (9)) [18].

In conjunction with the quantum probability theory above, a quantum walk model was used to mathematically describe characteristics [a], [g], and [h]. It was also used to analyze the dynamic behavior of the composite system created as a result of the interaction between multiple quantum fields. Furthermore, it was aimed at exploring the master equation for describing the dynamics of the DP by noting that their behaviors are similar to those of the quantum walk. It has been experimentally confirmed that these behaviors exhibited inherent characteristics that corresponded to those of the quantum walk [21]: The temporal behavior of the DP energy transfer between the two NPs in Fig.

3(a) was least-squares fitted to an exponentially decaying function $\exp(-t/\tau)$, where $t$ and $\tau$ represent time and the time constant of the phenomena, respectively. This exponential decay corresponds to the quantum walk dynamics*.

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(*) The temporal behavior of the random walk is represented by $\exp\left(-\sqrt{t/\tau}\right)$, which exhibits slower decay than that of the quantum walk.

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By referring to the arcsine law derived by the quantum probability theory, numerical simulations were carried out to analyze the creation of the DP and its energy transfer in a fiber probe-to-fiber probe system. As is schematically explained by Fig. 5(a), two fiber probes served as a sender and a receiver of the DP energy under collective excitation by conventional propagating light.

Two assumptions were made for this analysis. They were: (a1) The sender fiber probe was coherently excited by the incident light. (a2) The created DP hopped from one atom to an adjacent atom in a coherent manner, which corresponded to the quantum walk process. The analysis described three energy dissipation phenomena caused by the energy conversion from the DP to the conventional propagating light: (d1) The conversion to a conventional electromagnetic field to be guided backward to the main body of the sender fiber probe. (d2) The conversion to a conventional electromagnetic field to be guided forward to the main body of the receiver fiber probe. (d3) The conversion to a conventional electromagnetic field that propagates out from the tapered part of the fiber probe to the outer free space. As a result, it was confirmed that, among all of the created DPs, the one created by the pair of anti-parallel electric dipoles was localized at the tip of the fiber probe without being dissipated through phenomena (d1) – (d3).

Figures 5(b) and (c) show the calculated results for the single-tapered and double-tapered fiber probes, respectively [22]. They demonstrate that the double-tapered fiber probe concentrated the DP energy at its tip more efficiently than that at the single-tapered one. This suggests that the double-tapered fiber probe is more advantageous for creating/measuring the DP with higher efficiency, which is consistent with the experimental results [23].

Future developments in this study are expected to explain also characteristics [a], [g], and [h].

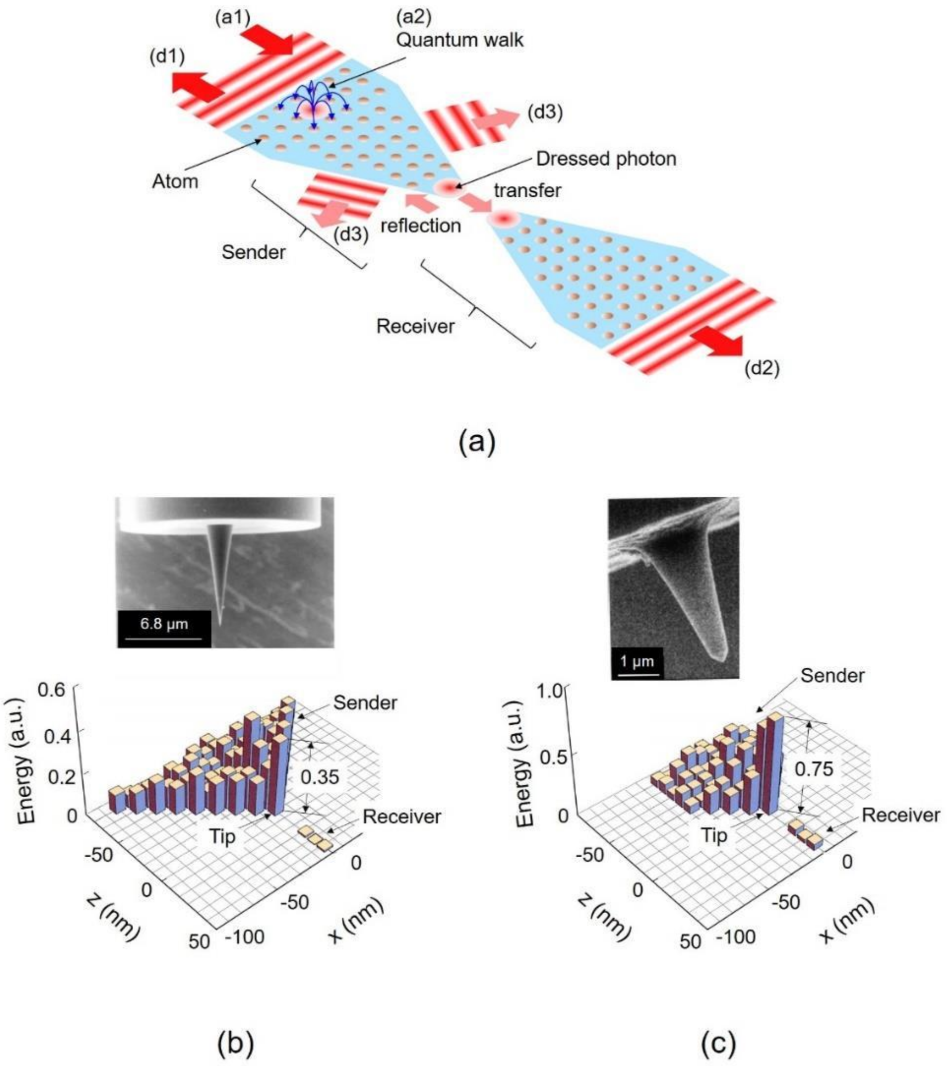

Fig.5 Simulation by a quantum walk model.

(a) The fiber probe-to-fiber probe system. (a1) and (a2) represent the two assumptions. (d1)-(d3) are the energy dissipation phenomena. (b) and (c) represent the calculated results for single-tapered and double-tapered fiber probes, respectively. The photos show scanning electron microscopic images of these fiber probes.

## 4.2 Other basic theories having a mathematical basis

**(1) Quantum measurement theory**: A theoretical description of characteristic [f] is essential for understanding the process of measuring the DP. Here, the problem is how to describe the dynamics of the DP energy transfer that occurs during the measurement. To solve this problem, quantum measurement theory, a branch of algebraic quantum theory, is under construction based on the theory of operator algebra, especially, $C^*$-

algebra. $C^*$-algebraic quantum theory is advantageous because it can explicitly describe macroscopic classical levels of quantum systems.

Mathematical issues for constructing the algebraic quantum measurement theory for the DP have been surveyed [24, 25]. They are:

1) Two methods are possible. Their mathematical issues are: [For the top-down method] After the mathematical model is built based on the universal gauge principle of quantum electrodynamics, several approximations should be made depending on the scale of the system or the properties of the material fields. [For the bottom-up method] This method is advantageous to build a mathematical model for describing the properties of the energy–momentum and the properties of localization of the DP. This model should be built by considering the ability to extend and scale it.

2) Mathematical modeling should start from the space-time area $\mathcal{O}$ in which nano-materials are provided. Here, a sub-space of the real space can work as the area $\mathcal{O}$. Next, an algebra $\mathcal{A}(\mathcal{O})$, composed of physical quantities in the area $\mathcal{O}$, is considered. Then, the temporal evolution $\alpha_t^{\mathcal{O}} \cap \mathcal{A}(\mathcal{O})$ is considered for each area $\mathcal{O}$. Microscopic physical quantities, representing the boundary conditions (the lattice defects, as an example), can be included in $\alpha_t^{\mathcal{O}}$. For this consideration, the measurement process can be represented by the inclusion relation $\mathcal{O} \subset \mathcal{O}_2$, where $\mathcal{O}_2$ represents the space-time domain under study. Finally, the measurement theory is expected to be established by the algebra $\mathcal{A}(\mathcal{O}_2)$.

**(2) Theory based on micro–macro duality**: Based on an algebraic quantum field theory, micro–macro duality theory has been constructed as a powerful mathematical guide for analyzing the nature of the DP [26]: Symmetry breaking in the algebra in a microscopic area can produce multiple sector spaces. Some physical quantities in these sector spaces satisfy the commutativity requirement, and the quantity named the center can be used to classify the sector spaces. That is, a commutative observable classical system and a non-commutative quantum system can coexist in each sector space, and this provides the basic structure for quantum-classical correspondence.

The sector space can be interpreted also as a mathematically symmetric space. It has been found through this interpretation that the automorphic form plays an essential role. Several discussions were made by taking a fiber probe as a test system: In order to construct a consistent theory for describing the DP, it will be a crucial breakthrough to faithfully reproduce its proper dynamic functions. This reproduction forms the micro–macro boundary level described by a symmetric space arising from a broken symmetry, which is possible by projecting the *s*-channel structure at the invisible micro-level to the spacelike *t*-channel. If suitable automorphic forms defined on this symmetric space are successfully identified, it will become possible to describe any of non-trivial dynamic phenomena caused by the DP. In particular, the automorphic factor appearing in the definition of an automorphic form will play an important role as a cocycle carrying the dynamic properties of the invisible micro-level. In the context of the DP, this will perhaps justify an analogy with the dynamic behavior played by the Regge trajectories, which carry spacelike momenta in the hadronic scattering processes originating from the dual resonance structure.

As is shown by Fig. 6, the theory based on micro–macro duality serves as a foundation of the theories reviewed in this article. This figure also summarizes the principal characteristics of the DP, the developed theories, their physical and/or mathematical methods, and information derived by these theories. The red double-pointed arrows indicate the topics commonly described by the multiple theories. By noting these arrows, correlations between the theoretical studies can be clearly recognized. Successful construction of off-shell science, guided by systematic studies on the DP, is expected by analyzing these correlations. It is also expected that the micro–macro duality theory will serve as a guide to this development.

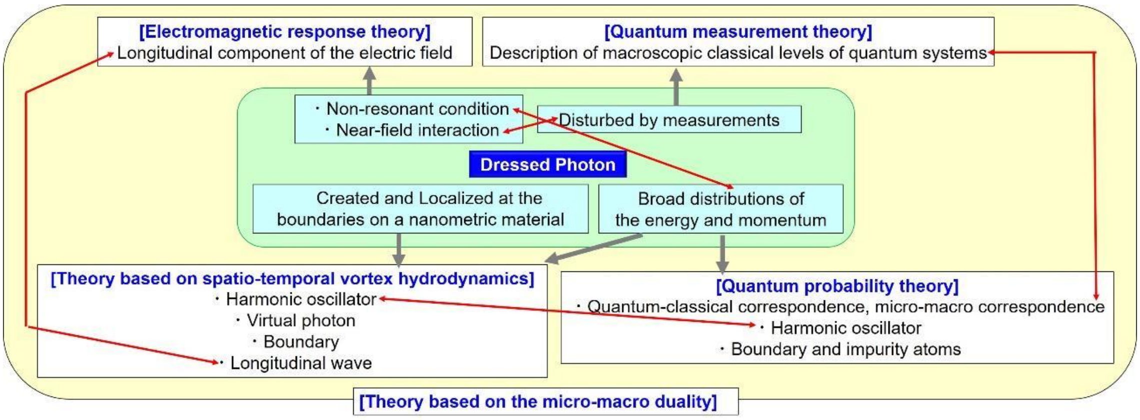

Fig.6 The principal characteristics of the DP, developed theories, their physical and/or mathematical methods, and information from the theoretical studies described in Sections 3 and 4.

Red double-pointed arrows indicate the topics common to the adjacent theories.

## 5. Summary

This article reviewed recent progress in the theoretical studies toward the development of off-shell science. First, it was pointed out that the DP is a quantum field whose energy–momentum relation deviates from the mass-shell. Second, the characteristics of the DP, as an off-shell quantum field, were reviewed. Third, theories having a physical basis were demonstrated. They were the electromagnetic response theory and a theory based on spatio-temporal vortex hydrodynamics. Fourth, theories having a mathematical basis were introduced, which can serve as invaluable guides for gaining a deep understanding of the concepts of the physics-based theories above. These theoreies were quantum probability theory and quantum measurement theory. Finally, a micro–macro duality theory was demonstrated, which serves as a foundation for embarking on the study of off-shell science.

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