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Off-shell science theories on interaction for dressed photons

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Abstract

This article, first, proposes a new approach to quantum fields in terms of category algebras and states on categories. Quantum fields and their states are respectively defined as category algebras and states on causal categories with partial involution structures. It is pointed out that, by utilizing category algebra and states on categories, relativity and quantumness can be directly integrated as a category theoretic structure and as a noncommutative probabilistic structure, respectively. Second, based on a Clebsch dual field theoretical model, it is made clear that a dressed photon (DP) field originates from a transition of the spacelike momentum of the Majorana field to a timelike one. This model derives a maximum size of the DP that has already been found by experimental studies. It is pointed out that, in the case where the paired timelike Majorana particle and anti-particle have anti-parallel spins, the pair annihilation creates a DP with a spin 0. The light converted from this DP can be a unique light field with spin 0, which behaves as a particle. It is experimentally confirmed that a cluster of photons emitted from an Si-LED behave as such a particle. Finally, a quantum walk model is employed to analyze the experimentally confirmed phenomena of DP energy transfer between nano-particles. Its theoretical bases are described based on the theory of category.

1. Introduction

It has been known that the dressed photon (DP) is a quantum field created as the result of the light–matter interaction that is induced by irradiating a nanometer-sized particle (NP) with light [1]. Thus, an interacting quantum field must be studied for constructing an accurate theorical model for the DP. However, the existence of a nontrivial interacting quantum field model defined on a four-dimensional Minkowski spacetime has not yet been proven. Axiomatic approaches to quantum field theories have derived many fundamental theorems, including the Haag theorem [2,3]. It is a no-go theorem, implying that an "interaction picture exists only if there is no interaction", through clarification of the concept of a quantum field [4,5]. To put it roughly, we cannot go beyond the theories for free fields if we stick to the axioms for conventional quantum field theories.

Intensive discussions on the theoretical methods based on classical Clebsch dual (CD) fields have been made in order to go beyond free fields, and a possible mechanism of DP creation has been made clear recently. These discussions have also succeeded in describing several experimental results by quantizing the DP energy [6]. Furthermore, a quantum walk (QW) model has been employed to analyze the spatio-temporal behaviors of the DP energy transfer.

Section 2 reviews the progress in theoretical studies based on category algebras, which serve as the bases of studying the DP [7-9]. Section 3 reviews the mechanism of DP creation based on CD fields. Section 4 reviews the relation between the QW model and the topics in Sections 2 and 3. Section 5 summarizes the discussions in this article.

2. Approaches based on category algebras

A new approach to quantum fields is proposed here to go beyond the free fields mentioned in the previous section [7-9]. The core idea is to deal with quantum fields in terms of noncommutative category algebra over a rig (ring without negatives). It is an algebraic system equipped with addition and multiplication, where the category and the rig correspond to the relativity aspect and the quantum aspect of nature, respectively. By utilizing category algebra and states on categories instead of simply considering categories, the two aspects above can be directly integrated as a category theoretic structure and as a noncommutative probabilistic structure, respectively. Through this integration, it is expected that quantum fields can be understood as the most fundamental entities in modern physics.

2.1 Treating relativity

Since the essence of relativity is nothing but the structure of possible relationships between possible events, what really matters are causal relationships [10]. For these relationships, there is an interesting order-theoretic approach to spacetime (for example, the causal set approach [11]). Furthermore, to deal with the off-shell nature of quantum fields, which seems to be essential in modelling interacting fields in space-time, one needs to take into consideration not only causal relationships but also more general relationships between spacelike events.

The strategy proposed here is to use a category C, which is a generalization of both ordered sets (causality structures) and groups (symmetry structures), as relativity in a generalized sense. More concretely, the notion of causal category equipped with a partial involution structure is identified as the generalized relativity structure.

2.2 Treating quantum fields

To combine the relativity structure above with quantum theory (which can be modelled by noncommutative rigs, effectively by noncommutative algebras over C), noncommutative algebras are required to reflect the structures of categories. Category algebras are just such algebras. They are generalized group algebras because categories are generalized groups.

Importantly, the category algebras can be considered as generalized matrix algebras over R as well as generalized polynomial algebras [9], which provides a platform for concrete and flexible studies and calculations. The category algebras have rich structure as covariance and local structure of subalgebras that reflect the causal and partial involution structure of the category. By focusing on these structures, one can also see the conceptual relationship between the present approach and the preceding approaches, such as algebraic quantum field theory [12,13] and topological quantum field theory [14,15].

After a quantum field is identified as a category algebra over a rig, the next problem is how to define a state of it. In general, the notion of state on *-algebra over C is defined as a positive normalized linear functional. The states on category algebras are called states on categories. More generally, defining a state on category (whose support is contained in a subcategory with finite numbers of objects) is equivalent to defining the corresponding function that assigns the weight to each arrow. By considering such states, a quantum mechanical system can be seen as an aspect of the quantum field.

For the study of quantum fields, a localized notion of state or a local state [16,17] is a key concept. The counterpart of the notion can be defined as the system of states on certain subalgebras of category algebras, called local algebras.

3 A novel theoretical model for dealing with interaction and longitudinal mode

As was pointed out at the beginning of Section 1, the DP is a localized quantum field whose size is much smaller than the wavelength of light. That is to say, the DP originates from an off-shell electromagnetic field. Furthermore, it should be pointed out that this field is associated with the longitudinal Coulomb mode, which plays an important role in light–matter interaction [6,18].

On the other hand, conventional propagating light is an on-shell electromagnetic field that can be observed in a macroscopic-sized region. It belongs to the visible sector and corresponds to a transverse mode photon. Conventional quantum electrodynamics have treated this mode, while the longitudinal mode has been excluded as an unphysical quantity. However, Ojima re-examined the quantization processes of the electromagnetic field and found that the longitudinal mode had to be included as a physically indispensable non-particle mode that plays an essential role in electromagnetic interaction [19,20]. This section reviews the novel theory constructed to deal with the interaction and the longitudinal mode [6].

3.1 Requiring spacelike momenta

Such a novel theory has to meet the requirement that has been stated by the Greenberg-Robinson (GR) theorem [21,22]. This theorem has been used to distinguish nonlinear field interactions from the free time evolutions of non-interacting modes. It states that *if the Fourier transform* $\phi(p)$

$\phi(x)$ *(p and x*

p_μ *with* $p_\nu p^\nu < 0$ (the sign convention of the Lorentzian metric signature $(+---)$ is employed), *then* $\phi(x)$ *is a generalized free field*. Although spacelike momenta are often associated with tachyons breaking Einstein causality, it is known that there exist certain types of causal motions having spacelike momenta.

The arguments above have revealed that spacelike momenta must be considered, while the classical longitudinal mode is closely related to virtual photons as the mediator of the longitudinal Coulomb force. More concretely, if the field is represented by a four-dimensional momentum p^{μ}, the GR theorem claims that not only the timelike and lightlike momenta ($p_{\nu}p^{\mu} \geq 0$) but also the spacelike momenta ($p_{\nu}p^{\mu} < 0$) are required for the interaction,.

Furthermore, the Haag theorem in Section 1 has claimed that such an interacting field (i.e., the Heisenberg field [23]) cannot be analytically connected to the propagating linear wave with unitary time evolution. Thus, it is a "no-go theorem" for the theoretical description of "interaction". In other words, no theories for "interaction" have ever existed so far.

3.2 Augmented electromagnetic theory for connecting to the spacelike region

The Maxwell equations can be expanded to the spacelike momentum region by using the CD field because this can introduce the longitudinal mode into the electromagnetic theoretical formulation. By this expansion, the conventional Maxwell equations can be analytically connected to the spacelike momentum region, and the longitudinal mode can be dealt with.

The lightlike CD field is represented by [1,6,18]:

$$\hat{T}_{\mu}^{\nu} = S_{\mu\sigma}S^{\nu\sigma} = \rho C_{\mu}C^{\nu}, \tag{1a}$$

$$\partial_{\nu}\hat{T}_{\mu}^{\nu} = 0, \tag{1b}$$

and

$$\rho \equiv L_{\tau}L^{\tau} < 0, \tag{1c}$$

where $S_{\mu\sigma}$ is the field strength. C_{μ} and L_{μ} are gradient vectors of the Clebsch variables that are introduced by

$$\partial_{\nu}\hat{T}_{\mu}^{\nu} = F_{\mu\nu}\partial_{\sigma}F^{\nu\sigma} = F_{\mu\nu}(-\partial^{\sigma}\partial_{\sigma}A^{\nu} + \partial^{\nu}(\partial_{\sigma}A^{\sigma})) = 0, \tag{2a}$$

$$\phi := \partial_{\sigma}A, \ \partial^{\tau}\partial_{\tau}\phi = 0, \ C_{\mu} := \partial_{\mu}\phi, \tag{2b}$$

$$\partial^{\tau}\partial_{\tau}\lambda-\left(\kappa_0\right)^2\lambda=0\,,\;\; L_{\mu}:=\partial_{\mu}\lambda\,,\;\; C^{\nu}L_{\nu}=0\,, \tag{2c}$$

and

$$U_{\mu}=\lambda C_{\mu}\,,\;\; S_{\mu\nu}=\partial_{\mu}U_{\nu}-\partial_{\nu}U_{\mu}=L_{\mu}C_{\nu}-C_{\mu}L_{\nu}\,. \tag{2d}$$

It should be noted that the term $\partial_{\sigma}A^{\sigma}$ in the Lagrangian

$$L^{*}=L+L_{GF}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}\left(\partial_{\nu}A^{\nu}\right)^2, \tag{3a}$$

and

$$(-\partial_{\sigma}F^{\nu\sigma}+\partial^{\nu}(\partial_{\sigma}A^{\sigma}))\delta A_{\nu}=0\,, \tag{3b}$$

introduced by Fermi, is identical to eqs. (2a) –(2d) if $F_{\mu\nu}\partial^{\nu}(\partial_{\sigma}A^{\sigma})=0$. That is, this term is identical to the classical longitudinal mode that follows the energy–momentum conservation law. Here, by referring to the Nakanishi-Lautrup (NL) formalism on the covariant quantization of the electromagnetic field [20], L_{GF} in eq. (3a) is expressed as

$$L_{GF}=B\partial_{\mu}A^{\mu}+\frac{\alpha B^2}{2}, \tag{4a}$$

$$\partial_{\mu}A^{\mu}+\alpha B=0\,, \tag{4b}$$

and

$$\partial^{\tau}\partial_{\tau}B=0\,. \tag{4c}$$

Since eqs. (3a) and (3b) above can be derived by setting $\alpha=1$ in eqs. (4a) and (4b), the Feynman gauge given by the NL formalism claims that $\partial_{\nu}A^{\nu}$ in eqs. (3a) and (3b) represents the longitudinal mode.

If the four-dimensional vector potential U_{μ} is re-defined by

$$U_{\mu}=\frac{1}{2}(\lambda C_{\mu}-\phi L_{\mu})\,, \tag{5a}$$

$$\partial^{\tau}\partial_{\tau}\lambda-(\kappa_0)^2\lambda=0\,, \tag{5b}$$

$$\partial^{\tau}\partial_{\tau}\phi-(\kappa_0)^2\phi=0\,, \tag{6a}$$

and

$$C^{\tau}L_{\tau}=0 \qquad (6b)$$

for expanding the electromagnetic field from the lightlike to spacelike region, the spacelike U_{μ} with $U^{\tau}U_{\tau}<0$ satisfies the equation $U^{\tau}\partial_{\tau}U_{\mu}=0$ that represents motion along a geodesic. Furthermore, the energy–momentum tensor $\hat{G}_{\mu}^{\nu}$, following the conservation law $\partial\nu\hat{G}_{\mu}^{\nu}=0$, satisfies

$$\hat{G}_{\mu}^{\nu}=-\hat{S}_{\mu\sigma}^{\nu\sigma}+\frac{1}{2}\hat{S}_{\alpha\beta}^{\alpha\beta}g_{\mu}^{\nu} \qquad (7a)$$

and

$$\hat{S}_{\mu\nu}^{\sigma\rho}\equiv S_{\mu\nu}S^{\sigma\rho}. \qquad (7b)$$

They are isomorphic to the Einstein equation of eqs. (8a) and (8b) below that are represented by the Riemann curvature tensor $R_{\mu\nu}^{\sigma\rho}$:

$$G_{\mu}^{\nu}=\kappa T_{\mu}^{\nu}, \qquad (8a)$$

and

$$G_{\mu}^{\nu}=-R_{\mu\sigma}^{\nu\sigma}+\frac{1}{2}R_{\alpha\beta}^{\alpha\beta}g_{\mu}^{\nu}. \qquad (8b)$$

By noting the relation between eqs. (7) and (8), and the "quantized space-time" introduced by Snyder [24], and furthermore, by noting the momentum vector

$$\hat{p}^{\nu}(\hat{p}_{\nu})^{*}=-\Lambda_{dS}=const.<0 \qquad (9)$$

in the de Sitter space, it is found that the quantity L_{ν} in the spacelike CD field is the submanifold in the de Sitter space.

If the momentum field satisfies eq. (9), space-time can be quantized, which is consistent with the Lorentz covariance. Therefore, the fact that eqs. (7) and (8) are isomorphic means that the quantization of the spacelike CD field is consistent with the space-time quantization. This means that the classical Maxwell equations were successfully expanded from the lighlike to spacelike momentum region.

3.3 Creation of Majorana field

It should be pointed out that the quantized field corresponding to the spacelike Klein-Gordon equation

$$(\partial^{\nu}\partial_{\nu}-(\kappa_0)^2)\lambda=0 \qquad (10)$$

is given by the Majorana field. To confirm this, let us consider the Dirac equation

$$(i\gamma^{\nu}\partial_{\nu}+m)\Psi=0, \qquad (11)$$

which can be regarded as the "square root" of the timelike Klein-Gordon equation ($(\partial^{\nu}\partial_{\nu}+m^2)\Psi=0$).

Thus, it can be readily seen that the Dirac equation for eq. (10) is expressed as $i(\gamma^{\nu}\partial_{\nu}+\kappa_0)\Psi=0$.

It has been reported that for eq. (11), there exists an electrically neutral Majorana representation in which all the components of the matrix γ take purely imaginary values such that it takes the form $((\gamma_{(M)})^{\nu}\partial_{\nu}+m)\Psi=0$. This equation clearly shows that a Majorana field λ satisfying $(\gamma^{\nu}\partial_{\nu}+\kappa_0)\lambda=0$ corresponds to the spacelike Klein-Gordon equation (eq. (10)). Reference [25] has explained how a couple of fermionic Majorana fields λ and φ (spin 1/2) can form a bosonic field (spin 1) that corresponds to the CD field. This explanation is based on Pauli`s exclusion principle and corresponds to the orthogonal condition $C^{\tau}L_{\tau}=0$ (eq. (6b)).

Another important aspect of the quantization is that the length (or wavenumber) must be quantized in the Majorana field [25]. This corresponds to the successful derivation of space-time quantization performed by Snyder [24], who worked on the spacelike momentum field defined on the de Sitter space. As a result, it is confirmed that the Majorana field was derived through this discussion of the CD field.

3.4 Mechanism of dressed photon creation

Since the DP field is created at a point-like singularity, let us consider a case in which a spacelike field λ in eq. (10) is perturbed by the interaction with a point source of the form $\delta(x^0)\delta(r)$, where x^0 and r respectively denote time and the radial coordinate of a spherical coordinate system. The solution can be expressed by the superposition of a spacelike stable oscillatory mode and a timelike unstable mode whose combined amplitude moves at a speed less than the velocity of light. A timelike unstable mode of the solution has the form $\lambda(x^0,r)=\exp(\pm k_0 x^0)R(r)$, where $R(r)$ satisfies

$$R''+\frac{2}{r}R'-(\hat{\kappa}_r)^2 R=0, \qquad (12a)$$

and

$(\hat{\kappa}_r) := (k_0)^2 - (\kappa_0)^2 > 0$. (12b)

Here, $R(r)$ is known as the Yukawa potential

$$R(r) = \exp(-\hat{\kappa}_r)/r \,, \tag{13}$$

which rapidly falls off as r increases.

A crucial kinematic property that distinguishes quantum mechanics from classical mechanics is the temporal directions of a moving particle and anti-particle. For an electrically neutral Majorana particle field, this property corresponds directly to time reversal, which means that a couple of unstable fields $\lambda(x^0, r) = \exp(\pm k_0 x^0) R(r)$ in the classical system can be reinterpreted as a particle and anti-particle pair in a quantum mechanical system. Thus, the argument above shows that a pair consisting of a timelike Majorana particle and an anti-particle pops up at the origin $r = 0$ as a result of field interactions between the field given by eq. (10) and $\delta(x^0)\delta(r)$. However, since these particle fields are non-propagating, the mechanism of pair annihilation would occur instantly to produce a small light field with a spatial distribution $R(r)$. This light field, which is the genesis of the DP, is a timelike quantum field and belongs to the visible sector. It should be noted again that such a small spatial DP field originates from a transition of the spacelike momentum of the Majorana field into a timelike one.

However, this small DP cannot be observed in the macroscopic area because it is much smaller than the wavelength of conventional propagating light. That is, its size is beyond the diffraction-limit for measurement. For measurement, the DP field must be disturbed to create free photons (propagating light) by inserting a probe into the DP field.

3.5 Maximum size of the dressed photon

Since a quantum mechanical discussion has shown that k_0 in $\exp(\pm k_0 x^0)$ and in eq. (12b) is the energy of a given system, the quantization of the Majorana field suggests that the wavenumber quantization with $Min[k_0] = \kappa_0$ $(, \Delta k_0 = \kappa_0)$ is advantageous because it is similar to the well-known energy quantization $E = h\nu$. Thus, it is found that this wavenumber quantization is valid for the quantum version of eq. (12b). That is, the quantized DP energy $E_{DP,n}$ is expressed as $E_{DP,n} \propto (\kappa_r)^2 = n(\kappa_0)^2$, where $n(=1,2,3,---)$ is a quantum number.

The advantage of the radial distribution given by $R(r)$ (eq. (13)) shows that this distribution has a clear-cut minimum value κ_0 of $\hat{\kappa}_r$ (refer to eq. (12b)). The existence of such a minimum wavenumber κ_0 means that there exists a maximum size of the DP defined by

$$L_{\max}^{DP} := \frac{1}{\kappa_0} \tag{14}$$

whose value has been evaluated experimentally to be 40–70 nm [26]. Here, $L_{\max}^{DP}$ is called the DP constant [6,18]. From the viewpoint of a new natural unit system in which all the magnitudes of the Planck constant h, light velocity c, and $L_{\max}^{DP}$ are set to unity, the importance of this length has been discussed by Sakuma and Ojima [27]. They showed that $L_{\max}^{DP}$ gives the geometrical mean of the smallest Planck length and the largest length associated with a newly modified cosmological constant, related to their dark energy model defined by the ground state of a spacelike Majorana field and to their novel dark matter model defined solely by the Weyl conformal tensor field, respectively.

3.6 Conversion from dressed photon to bullet-like propagating light

In the case of a pair annihilation involving anti-parallel spins, the created DP has a spin 0. Specifically, the discussions of the DP in the preceding sections suggest the possibility that the light converted from the DP can be a unique light field with spin 0, which behaves as a particle. This particle-like behavior has been supported by the Wightman theorem [28] stating that: *A Lorentz or Galilei covariant massive system is always localizable. For the Lorentz case, the only localizable massless elementary system (i.e., irreducible representation) has spin 0.* Here, localizability means that a position operator can be defined for this system.

The lightlike CD field can be described by the system of eqs. (1) and (2), in which the field strength $S_{\mu\nu}$ is given by bivectors C_ν and L_ν, that satisfy the orthogonality condition $C_\nu L_\nu = 0$ (eq. (6b)). In this case, it is assumed that L_ν is a spacelike vector. Notice, however, that the orthogonality condition $C_\nu L_\nu = 0$ is also satisfied in the case of $L_\nu = C_\nu$ since C_ν is a null vector. Of course, in this case, the vortical field strength $S_{\mu\nu}$ vanishes. Recall that, quantum mechanically, the C_ν field is a Majorana field with spin 1/2; thus, a couple of anti-parallel C_ν fields with spins 1/2 and $-1/2$ can be combined to yield a null energy–momentum current $C_\mu C^\nu$ ($\hat{T}_\mu^\nu = \rho C_\mu C^\nu$ in eq. (1a)) with spin 0, which can be regarded as a unique bullet-like light field with spin 0.

3.7 Experimental verification

This subsection reviews results of experiments that were carried out to verify the behaviors of emitted light described in the previous section [6]. As background information for the present experiment, Wada et al. [29] showed that a silicon-LED (1.3–1.6 μm wavelength), successfully fabricated by a novel fabrication technology named DPP-assisted annealing, worked as a relaxation oscillator upon the injection of direct current to emit an optical pulse train whose duration and repetition frequency were about 50 ps and 1 GHz, respectively.

The experiments were carried out by following the well-known Hanbury Brown-Twiss method [30]. A similar experiment has been carried out to check the behavior of a single photon in a nanometer-sized semiconductor logic gate whose signal was controlled by using DPs [31]. For the present experiment, highly sensitive superconducting single-photon detectors were used to measure the temporal behavior of the infrared light emitted from small light sources in the LED.

The values of the second-order cross-correlation coefficient (CC) were measured as a function of the difference τ between the arrival times of the photons at the two independent photon detectors. The measured results showed two features: One was that the value of CC was smaller than unity in the range of time difference $|\tau| < 20$ ns. This indicates the photon anti-bunching phenomenon, which is an inherent feature of a single photon. The other was that the CC took a nonzero value at $\tau = 0$ even though it is smaller than 1×10^{-2}. This nonzero value is attributed to the photons emitted from multiple light sources located in close proximity with each other in the LED.

These two features suggest that a cluster of photons emitted from the LED behaves as a single photon. It is named DP-cluster light and is closely related to the localizable property of the spin 0 particle. Namely, if the observable positions of given spin 0 quantum particles are sufficiently close, the cluster of these particles would behave as if it were a single quantum particle with the accumulated amount of energy.

The original motivation of embarking on the DP study was to generate a diffraction-free small light field. However, the experimental verification above gives another suggestion, that is, such a peculiar propagating light field exists, whose energy-momentum tensor has exactly the same form as a free particle. If that is the case, a light beam consisting of such a light field would behave as a bullet and be free from diffraction. It is well known that a laser beam employing conventional transverse light waves is unavoidably diffracted. Although there exists a certain class of diffraction-free mode-solutions [32] for transverse light waves, these solutions should not be confused with the above-mentioned peculiar light field that is intrinsically diffraction-free. In regard to this peculiar light field, it is further conjectured that the mechanism of DP-cluster light may be involved in γ -ray bursts, one of the cosmological enigmas, as an intermittent extremely high-energy radiation with strong directionality that reaches the earth after travelling over an enormous distance of several billions of light years.

4. Quantum walk model for the energy transfer of dressed photon

Experimental studies have confirmed that the DP, created on one NP by light irradiation, hops to the adjacent NP. That is, the DP energy transfers from one NP to the adjacent one. This transfer originates from the interaction between the NPs mediated by the DP.

Creation and annihilation operators for the DP are required to describe this transfer. However, conventional quantum field theories have never succeeded in deriving them. This is because the DP is an interacting quantum field, for which the interaction is a nonlinear event. Conventional creation and annihilation operators have been derived only for the linear system. As long as these operators are not derived for the nonlinear interacting field, the origin of the pair-annihilation of the aforementioned Majorana particle and anti-particle will remain unknown.

However, since the created DP is a timelike boson field and since it spatially localizes at the NP, it is expected that experimentally confirmed spatio-temporal behaviors of the DP energy transfer can be described by using a QW model. A QW model for a quasi-particle, whose behavior is described by the Klein-Gordon equation, has been studied [33]. With future progress in this study, it is expected that the theory of DP creation described in the previous section and the QW model for DP energy transfer can be connected consistently. Furthermore, it is expected that this connection will enable the construction of a novel theory that can systematically describe the creation, energy transfer, and detection of the DP to identify the origins of a variety of experimentally confirmed phenomena.

Concrete dynamics of quantum fields can be modeled as a sequence or flow of the states on a category. A QW is a typical example of describing such dynamics [34,35]. The notion of a QW on general *-algebras and quantum walks on +-categories can be defined as follows: Let *A* be a *-algebra. A sequence of states given by– $\varphi^t(\alpha) = \varphi((\omega^*)^t \alpha \omega^t)$ where $t = 0,1,2,3,...$ generated by a unitary element $\omega \in R[C]$, i.e. an element satisfying $\omega^* \omega = \omega \omega^* = \varepsilon$, is called a quantum walk on *A*.

A QW can be considered as a sequence of state vectors through a Gel'fand-Naimark-Segal (GNS) construction. Numerical calculations have been embarked on [36] to analyze the DP energy transfer phenomena. It should be noted that the DP cannot be understood without focusing on the off-shell nature of quantum fields [37]. This means that the aspects of quantum fields cannot be described as a collection of the modes which satisfies the on-shell condition, and that a QW on categories may become important in quantum field theory.

5. Summary

A new approach to quantum fields was proposed in terms of category algebras and states on categories. Quantum fields and their states were respectively defined as category algebras and states on causal categories with partial involution structures. It was pointed out that, by utilizing category algebra and states on categories, relativity and quantumness can be directly integrated as a category theoretic structure and as a noncommutative probabilistic structure, respectively.

Based on a CD field theoretical model, it was made clear that a DP field originates from a transition of the spacelike momentum of the Majorana field to a timelike one. This model succeeded in deriving a maximum size of the DP that has been already found by experimental studies. This size was named the DP constant. It was found that, in the case where paired timelike Majorana particle and anti-particle have anti-parallel spins, pair annihilation creates a DP with a spin 0. The light converted from this DP can be a unique light field with spin 0, which behaves as a particle. It was experimentally confirmed that a cluster of photons emitted from an Si-LED behaved as such a particle and is named DP-cluster light.

A QW model was employed to analyze the experimentally confirmed phenomena of DP energy transfer between NPs. Its theoretical bases were reviewed based on the theory of category.

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