

# Optimum dissipation for governing the autonomous transfer of dressed photons

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## Abstract

This paper claims that the unique features of dressed photon (DP) transfer are governed by the DP energy dissipation. First, problems on theoretical descriptions of DP creation, transfer, and measurement are presented, and strategies for solving them are also indicated. Second, experimental results of DP measurement are reviewed. It is pointed out that these results follow a principle that differs from the on-shell scientific principle of least action. Third, in order to analyze these results, a theoretical non-unitary quantum walk model is presented by considering energy dissipation. Finally, calculated results are presented, suggesting that an optimum dissipation constant of the dressed-photon-phonon energy exists. It is also claimed that the transfer path with such optimum dissipation is autonomously determined to minimize the decreases in the emitted light power. In other words, this determination is governed by the off-shell scientific principle of largest output signal.

## 1. Introduction

A large number of novel phenomena that originate from the nature of the dressed photon (DP) have been experimentally found and applied to realize innovative technologies [1]. As an introduction to this paper, this section summarizes the principles of DP creation, transfer, and measurement [2], as follows:

**[1] Creation** (Fig. 1(a)): A DP is created on a nanometer-sized particle (NP) by the interaction between a light field and a microscopic material field.

**[2] Transfer** (Fig. 1(b) and (c)): If a second NP ( $NP_2$ ) is placed in close proximity to the first NP ( $NP_1$ ) (Fig. 1(b)), the DP on  $NP_1$  transfers to  $NP_2$ . This transfer between these NPs is bi-directional because it originates from the interaction between the NPs. Even when the number of NPs is increased to  $N$  (Fig. 1(c):  $NP_1$  to  $NP_N$ ), bi-directional transfer of the DP also occurs.

**[3] Measurement** (Fig. 1(d)): For measuring the DP, a larger-sized NP ( $NP_L$ ) is placed at the end of

an array of smaller-sized NPs ( $NP_S$ ) [3]. After the DP reaches  $NP_L$  by repetitive bi-directional transfers between the adjacent  $NP_S$ s, an exciton is created in a higher energy level in  $NP_L$ . A part of its energy dissipates to the heat-bath, and the exciton relaxes to the lowest energy level. Subsequently, the exciton annihilates within a short time, resulting in propagating light emission, which is the DP energy dissipation to the external macroscopic space. This propagating light can be measured in the macroscopic space by using a conventional measurement instrument.

This paper claims that DP transfer is governed by the dissipation in the measurement process of [3]. Section 2 reviews the problems that must be solved for giving theoretical descriptions of DP creation, transfer, and measurement, and also presents strategies for solving them. Section 3 reviews the experimental results of DP measurements and indicates that DPs have unique features that do not follow the conventional on-shell scientific principle. For analyzing these features, Section 4 proposes a method of introducing the concept of dissipation into the theoretical quantum walk (QW) model. Section 5 describes the theoretical results and compares them with experimental ones. Section 6 presents a summary.

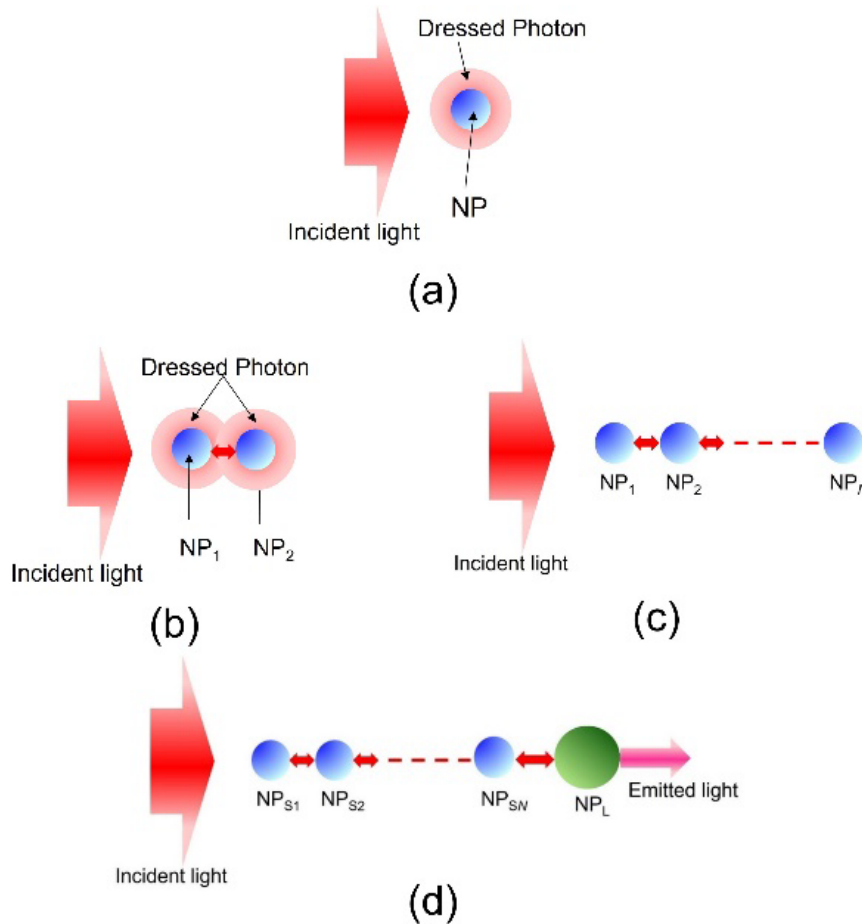


Fig. 1 Dressed photon (DP) creation, transfer, and measurement.

- (a) DP creation. (b) Bi-directional transfer of DP between two nanometer-sized particles (NPs).
- (c) Bi-directional transfer, where the number of NPs in (b) is increased from two to  $N$ .
- (d) DP measurement.

## 2. Problems and strategies for solving them

Basic problems in the modern science have to be solved for theoretically describing DP creation, transfer, and measurement indicated in Fig. 1 in a unified manner. These problems, strategies for solving them, and the solutions derived so far by novel off-shell scientific methods are summarized as follows:

**[1] Creation:** Since the DP creation in Fig. 1(a) originates from the light–matter interaction in a microscopic space, a spacelike momentum field must be introduced into the electromagnetic field theory. Although conventional on-shell science has never succeeded in doing so, off-shell science has recently succeeded in drawing physical pictures of the creation process. These successful pictures are [4,5]:

- (1) A microscopic material field (a timelike momentum field) interacts with a vector boson field.
- (2) In addition to a stable pair of Majorana fermions (a spacelike momentum field), an unstable pair consisting of a Majorana particle and anti-particle (timelike momentum fields) is created (pair creation).
- (3) Although this unstable pair annihilates within a short time duration (pair annihilation), a novel light field (a timelike momentum field) remains at the microscopic material. This is the DP.

**[2] Transfer:** Since DP transfer is a dynamic process in a microscopic complex system, the concept of interaction between NPs via exchange of a DP must be introduced into the theoretical model, and the position of the DP must be identified (Figs. 1(b) and (c)). Recent studies have found that a QW is a promising theoretical model for this introduction and identification [6-11]. This is because the principle of the QW and the nature of the DP have at least two common features:

**(a) Nonreciprocity:** A mathematical formulation of the QW uses nonreciprocal algebra that is composed of vectors and matrices. On the other hand, as has been described in [1] above, the DP is a field that originates from the interaction between NPs. Since the interaction is a typical nonreciprocal physical process, the QW and DP have a common feature, represented by nonreciprocity. This allows the QW to describe the interaction and the bi-directional DP transfer.

**(b) Site:** The QW deals with the phenomenon of the energy transfer from one site to its neighbor. On the other hand, since the DP is spatially localized<sup>(\*)</sup>, its quantum mechanical position operator can be defined. Thus, in the case where the site of the QW is the NP on which the DP is created, the position of the DP can be identified as the position of this site.

**[3] Measurement:** The DP transfer in Figs. 1(b) and (c) can neither be monitored nor measured in a macroscopic space because it occurs only in a microscopic space. For measurement, the DP energy must be delivered from the microscopic to the macroscopic space via energy dissipation (Fig. 1(d)). By introducing this dissipation into QW theory, it is transformed from unitary to non-unitary. The dissipation at the NP<sub>SS</sub> in a microscopic space plays a leading role in this delivery. The main purpose of this paper is to describe this role.

Table 1 summarizes the reviews [1] - [3] above. The concepts and strategies in this table are correlated among [1] - [3].

Table 1 Concepts required for describing the DP creation/transfer/measurement and strategies for description

	[1] Creation	[2] Transfer	[3] Measurement
Phenomena and spaces	Light–matter interaction in a microscopic space	Dynamic behavior of a complex system in a microscopic space	Energy transfer from microscopic to macroscopic space
Concepts	Interaction	Nonreciprocity and sites	Dissipation
Strategies	Introducing the spacelike momentum field	Using a unitary QW model	Using a non-unitary QW model

(\*) The theory of DP creation teaches that the spin of the electric DP is zero [4,5], which has also been experimentally confirmed. Such a zero-spin field is spatially localized, as indicated by Wightman's theorem [12].

### 3. Experimental results

This section reviews the experimental results in Fig. 1(d) [2]. As is schematically explained by Fig. 2(a), the experiments used a three-dimensional arrangement of a large number of small NPs ( $NP_{SS}$ ) dispersed in a box-shaped template. A large NP ( $NP_L$ ) is placed on the top of this box and is used as an output port [13]. Since the semiconductor CdSe NP used for the experiments absorbs some amount of the light power, the DP energy dissipates and decreases by repeating the DP transfer between adjacent NPs. Thus, the propagating light power emitted from  $NP_L$  decreases by increasing the number of  $NP_{SS}$  that are dispersed between the input and output ports (by increasing the direct distance  $L$  between these ports). The rate of the decrease due to the internal dissipation above is formulated as  $\exp(-\alpha_{loss}x)$ , where  $\alpha_{loss}$  is the absorption coefficient and  $x$  is the optical path length.

Figure 2(b) shows the experimental results. The heights  $H$  of the template boxes of the specimens A–C were 10 nm, 20 nm, and 50 nm, respectively. They are proportional to the number  $N_z$  of  $NP_{SS}$  that were vertically piled up. This figure shows the relation between the direct distance  $L$  and the light power emitted from  $NP_L$ . Broken lines represent the exponential functions fitted to the measured values. Since the output light power, normalized to the input power, is the transmittance  $T$ , this function is expressed as  $T = \exp(-L/L_0)$ , where  $L_0$  is named the attainable distance.

Figure 2(c) shows the relations between  $H$  and  $L_0$  for the specimens A–C. They indicate a monotonic increase in  $L_0$  with an increase in  $H$ . Since the  $NP_{SS}$  in the template box enable the formation of a longer path for the DP transfer by increasing  $H$ , decreases in the emitted light power are expected by this increase due to the increase in the amount of internal dissipation over the full

length of the path. However, the monotonic increase in Fig. 2(c) is contrary to this expectation. These unexpected results imply the intrinsic off-shell scientific features. They are: The DP selects a path that minimizes the decreases in the emitted light power. In other words, the DP autonomously finds **the** path so that it delivers the highest energy to the macroscopic system. This path is not necessarily the shortest of all the geometrically feasible paths in the box. That is, the principle for the path selection is different from the that of the on-shell scientific principle of least action.

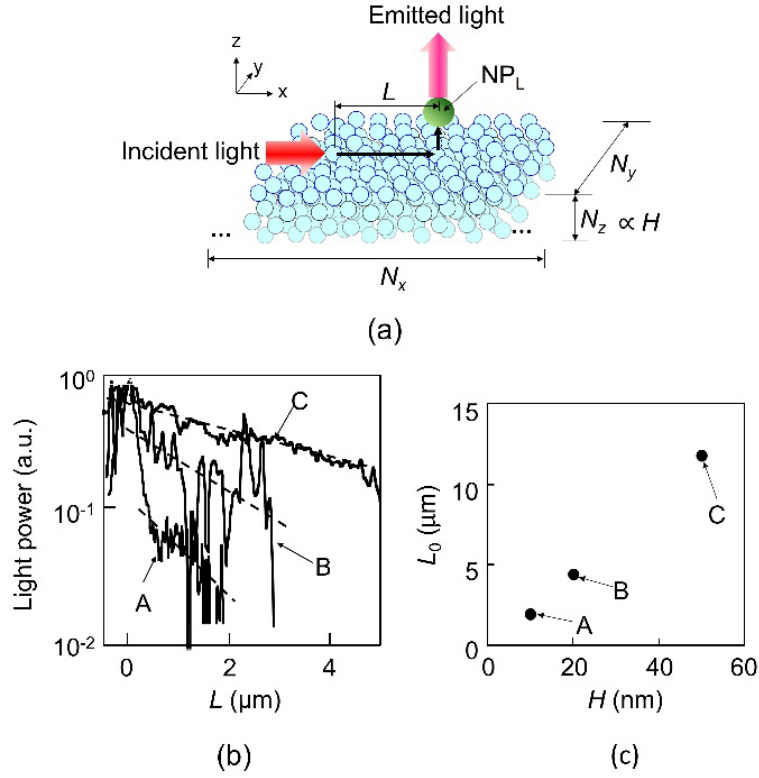


Fig. 2 Bi-directional DP transfer in a large number of small NPs (NP<sub>ss</sub>) and propagating light emitted from the large NP (NP<sub>L</sub>).

(a) Schematic explanation of box-shaped arrangement. A semiconductor CuCl was used as the NP material.

(b) Measured relation between the direct distance  $L$  and the output light power emitted from NP<sub>L</sub>. The heights  $H$  of the three-dimensional arrangements of NP<sub>ss</sub> of the specimens A-C were 10 nm, 20 nm, and 50 nm, respectively. Broken lines represent the exponential functions  $\exp(-L/L_0)$  fitted to the measured values, where  $L_0$  is the attainable distance.

(c) Relation between  $H$  and  $L_0$  for the specimens A-C.

#### 4. Theoretical model

Noting that interaction and dissipation are indispensable for describing the unique features of the DP [14], the QW has been used as a promising theoretical model for analyzing the DP transfer ([2] in Section 2) [6-10]. The contribution of dissipation was added to this model for deriving the probability

of the dressed-photon–phonon (DPP) created at the B atom-pair in a silicon (Si) crystal [11]. This section reviews the formulation of this model by starting from the case without energy dissipation.

By using of the light incident to the lower side of a two-dimensional lattice assumed for this model, a DP is created at each site in this lattice and travels in the upper or lower directions. During these travels, the DP repeats hopping from a site in the lattice to its nearest-neighbor site. The phonon does not hop due to its nonlocalized nature. Since the DPP is created as a result of coupling between two counter-travelling DPs and a phonon, a three-dimensional vector

$$\vec{\psi}_{t,(x,y)} = \begin{bmatrix} y_{DP+} \\ y_{DP-} \\ y_{Phonon} \end{bmatrix}_{t,(x,y)} \quad (1)$$

is used to represent its creation probability amplitude, where  $[ \ ]$  is the vector at time  $t$  and at the position  $(x, y)$  of the lattice site,  $y_{DP+}$  and  $y_{DP-}$  are the creation probability amplitudes of the DPs that travel by repeating the hopping in the upper or lower directions, respectively, and  $y_{Phonon}$  is that of the phonon.

For representing spatial-temporal evolution equations for the DPP that hops out from the site, the vectors

$$\left[ \vec{\psi}_{t,(x,y)\leftrightarrow} \right] \equiv \begin{bmatrix} y_{DP+\rightarrow} \\ y_{DP-\leftarrow} \\ y_{Phonon} \end{bmatrix}_{t,(x,y)} \quad (2a)$$

and

$$\left[ \vec{\psi}_{t,(x,y)\Downarrow} \right] \equiv \begin{bmatrix} y_{DP+\uparrow} \\ y_{DP-\downarrow} \\ y_{Phonon} \end{bmatrix}_{t,(x,y)} \quad (2b)$$

are used. In eq. (2a), the vector  $\vec{\psi}_{t,(x,y)\leftrightarrow}$  represents the DPP, hopping out from the site, which is composed of two DPs ( $y_{DP+\rightarrow}$  and  $y_{DP-\leftarrow}$  that hop along the  $\pm x$  axes) and a phonon ( $y_{Phonon}$ ) at time  $t$ . In eq. (2b), the vector  $\vec{\psi}_{t,(x,y)\Downarrow}$  represents the DPP, which is composed of two DPs ( $y_{DP+\uparrow}$  and  $y_{DP-\downarrow}$  that hop along the  $\pm y$  axes) and a phonon ( $y_{Phonon}$ ).

By using eqs. (2a) and (2b), the spatial-temporal evolution equation for the DPP is represented by

$$\vec{\psi}_{t,(x,y)} = \begin{bmatrix} \vec{\psi}_{t,(x,y)\leftrightarrow} \\ \vec{\psi}_{t,(x,y)\downarrow} \end{bmatrix} = \begin{bmatrix} [0] & U \\ U & [0] \end{bmatrix} \begin{bmatrix} \vec{\psi}_{t-1,(x,y)\leftrightarrow} \\ \vec{\psi}_{t-1,(x,y)\downarrow} \end{bmatrix}, \quad (3)$$

where

$$U \equiv \begin{bmatrix} \varepsilon_+ & J & \chi \\ J & \varepsilon_- & \chi \\ \chi & \chi & \varepsilon_0 \end{bmatrix} \quad (4)$$

and

$$[0] \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

In eq. (4),  $U$  is a unitary matrix whose diagonal elements  $\varepsilon_+$  and  $\varepsilon_-$  are the eigen-energies of the DPs ( $y_{DP+}$  and  $y_{DP-}$ ), respectively, and  $\varepsilon_0$  is that of the phonon. Off-diagonal elements  $J$  and  $\chi$  represent the DP hopping energy and the DP-phonon coupling energy, respectively.

Next, the energy dissipation is introduced into this model [11]: The vector of eq. (1) is replaced by a four-dimensional vector

$$\vec{\psi}'_{t,(x,y)} = \begin{bmatrix} y_{DP+} \\ y_{DP-} \\ y_{Phonon} \\ y_{dis} \end{bmatrix}_{t,(x,y)}. \quad (6)$$

The fourth line  $y_{dis}$  represents the creation probability amplitude of the DPP dissipated from the lattice. The spatial-temporal evolution equation is represented by

$$\vec{\psi}'_{t,(x,y)} = \begin{bmatrix} [\vec{\psi}'_{t,(x,y)\leftrightarrow}] \\ [\vec{\psi}'_{t,(x,y)\downarrow}] \\ [\vec{\psi}''_{t,(x,y),dis}] \end{bmatrix} = \begin{bmatrix} [0] & \sqrt{1-\kappa^2}U & [0] \\ \sqrt{1-\kappa^2}U & [0] & [\kappa] \\ [0] & [\kappa] & [0] \end{bmatrix} \begin{bmatrix} [\vec{\psi}'_{t-1,(x,y)\leftrightarrow}] \\ [\vec{\psi}'_{t-1,(x,y)\downarrow}] \\ [\vec{\psi}''_{t-1,(x,y),dis}] \end{bmatrix}. \quad (7)$$

The third line of the left-hand side vector corresponds to the dissipated DPP

$$[\vec{\psi}''_{t,(x,y),dis}] \equiv \begin{bmatrix} y_{DP+,dis} \\ y_{DP-,dis} \\ y_{Phonon} \end{bmatrix}_{t,(x,y)}. \quad (8)$$

The first and second lines ( $y_{DP+,dis}$  and  $y_{DP-,dis}$ ) in eq. (8) represent the dissipated DPs that travel along the upper or lower directions in the lattice, respectively. Their sources are  $y_{DP+}$  and  $y_{DP-}$  in eq. (6), respectively.

The quantity  $\kappa$  in the matrix

$$[\kappa] \equiv \begin{bmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

on the right-hand side of eq. (7) is a phenomenologically introduced dissipation constant ( $0 \leq \kappa \leq 1$ ). As a result of introducing the matrix of eq. (9) into eq. (7), the coefficient matrix on the right-hand side of eq. (7) is transformed from unitary to non-unitary. The DPP energy stored in the lattice decreases as a result of the dissipation. The quantity  $\sqrt{1-\kappa^2}$  represents the magnitude of the energy left in the lattice after dissipation.

Numerical calculations derived the values of the creation probabilities of DPP at the B atom-pairs in Si-light emitting devices and the light power emitted from the Si crystal to the external macroscopic space [11]. The calculated results agreed well with the experimental results. The remarkable finding revealed by these calculations is that there exists an optimum value  $\kappa_{opt}$  (=0.2) of the dissipation constant  $\kappa$  that maximizes the emitted light power, as is shown in Fig. 3.



The results shown in Fig. 3 give a clue for analyzing the intrinsic features of the experimental results of Fig. 2(d) when the B atom-pair in a Si crystal is replaced by NPs: After this replacement, the dissipation constant  $\kappa$  is given by the sum ( $\kappa = \kappa_{\text{in}} + \kappa_{\text{out}}$ ) of the internal dissipation constant  $\kappa_{\text{in}} (= N\alpha_{\text{loss}})$  at all the NPs ( $N$ : number of NPs in the box) and the dissipation constant  $\kappa_{\text{out}}$  at the output port NP<sub>L</sub>. Since  $N$  is the product of the numbers  $N_x$ ,  $N_y$ ,  $N_z$  of the NPs along the  $x$ -,  $y$ -, and  $z$ -axes in the box and the height  $H$  is proportional to  $N_z$ , it is easily found that  $\kappa$  is proportional to  $H$ . This proportional relation implies that there exists an optimum value  $H_{\text{opt}}$  of  $H$  that maximizes the value of  $L_0$  in Fig. 2(c).

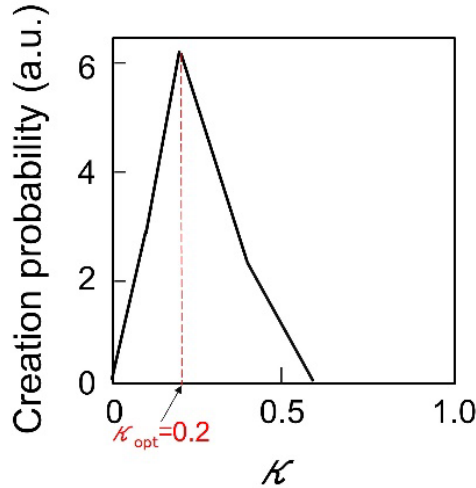


Fig. 3 Relation between the dissipation constant  $\kappa$  and the DPP creation probability at the B atom-pair.

## 5. Comparison with experimental results

Figures 4(a) and (b) are copies of Figs. 2(c) and 3, respectively. A comparison between them indicates that the monotonically increasing experimental values in Fig. 4(a) correspond to the calculated values in the area A ( $H < H_{\text{opt}}$ ) in Fig. 4(b). This correspondence also indicates that the value of  $L_0$  takes the maximum at  $H = H_{\text{opt}}$  (the optimum path) and subsequently decreases with increasing  $H$  (the area B). This means that the optimum transfer path above is autonomously determined, resulting in minimizing the decreases in the emitted light power. For this path, the internal dissipation constant

$\kappa_{\text{in}} (= N\alpha_{\text{loss}})$  plays a leading role ([3] in Section 2) because the optimum path is realized by optimizing the constant  $\kappa$  to  $\kappa_{\text{opt}}$ . Furthermore, since the value of  $H_{\text{opt}}$  is finite, the optimum path is composed of a finite number of NP<sub>ss</sub>. Finally, it is claimed that the features of the optimum path are intrinsic to off-shell science. In other words, this path is governed not by the on-shell scientific principle of least action but by the off-shell scientific principle of so-called largest output signal.

Future problems include how to describe  $H_{\text{opt}}$  in the formula and to identify the origin of the phenomenological dissipation constant  $\kappa$ .

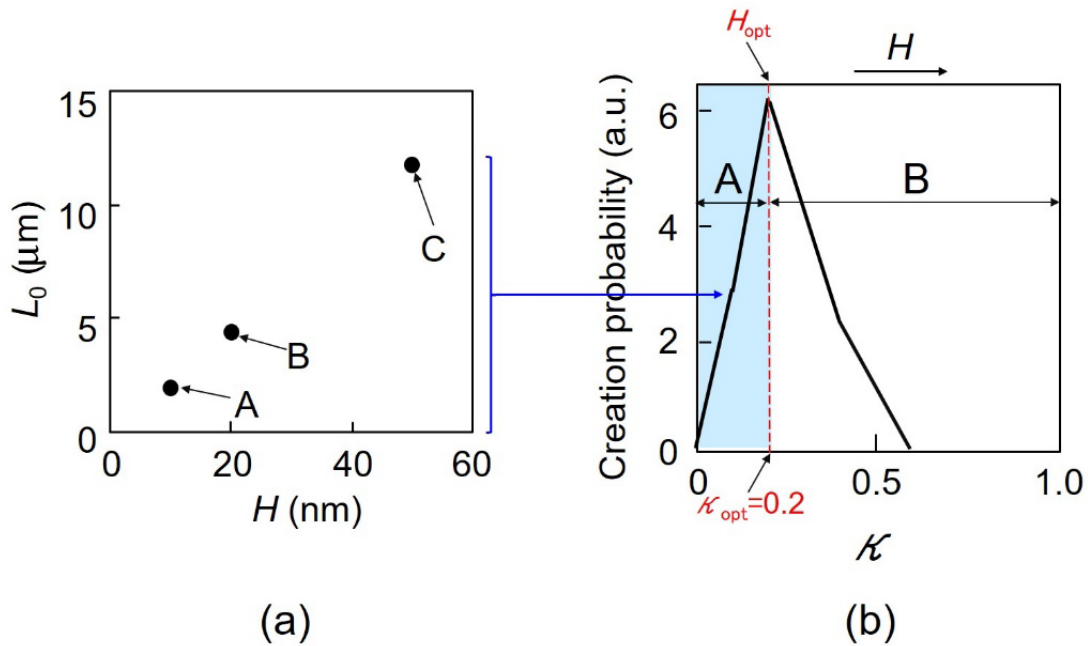


Fig. 4 Copies of Figs. 2(c) and 3 for comparing experimental and calculated results.

(a) Copy of Fig. 2(c). (b) Copy of Fig. 3.

## 6. Summary

This paper claimed that an optimum dissipation constant of DPP energy exists and that the transfer path with such optimum dissipation is autonomously determined to minimize the decreases in the emitted light power. In other words, this determination is governed not by the on-shell scientific principle of least action but by the off-shell scientific principle of largest output signal. These claims imply that there exists an optimum size of the three-dimensional arrangement of the NPs when it is applied to fabricate a novel nanometer-sized optical device. Future problems include how to describe

$H_{\text{opt}}$  in formula and to identify the origin of the dissipation constant  $\kappa$ .

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