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Drastic advancement in nanophotonics achieved by a new dressed photon study

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Abstract

On the very recent advancement of dressed photon studies: A timely review of the emerging new phase of dressed photon (DP) studies, not yet prevailing in the global nanophotonic society, is given in contradistinction to its preceding incipient phase. A new theory on DPs crucially depends on a couple of important elements, namely, the knowledge on quantum field theory (QFT) having infinite degrees of freedom, notably on the micro-macro duality (MMD) theory developed by Ojima, and a newly proposed Clebsch dual (CD) electromagnetic field as a specific implementation of MMD theory. The main aim of the first part of this article after the introduction, the section of "In search of refinement of the theoretical models", is twofold: to explain plainly, without resorting to mathematical equations, the essence of the highly mathematical contents of MMD theory, which clarifies a problematic aspect of the Schrödinger's cat thought experiment, and to explain the physical meanings of the CD field.

Preliminary study on the existence of DP light with spin zero: In the section of "Observed "photon cluster" and light field with spin zero", we briefly report a new intriguing experimental discovery implying the existence of propagating particle-like "quantum DP light" together with a conjecture on its possible theoretical explanation. A perspective on a variety of possible research directions for DPs is then briefly mentioned in the final section.

Keywords: Dressed photon, Micro-macro duality, Clebsch dual field, Majorana field, Dressed photon constant, Natural unit system

Introduction: overview of incipient dressed photon studies

Reflecting the demands of nano-technology at the time, a significant surge in near-field optics worldwide began around 1980, in which the third author M.O. was deeply involved, with the seemingly outrageous resolution of *generating small light free from the diffraction*, which would enable us to achieve a variety of high-performance technologies such as higher-density light recording, light processing and optical devices with higher resolution. The surge stemmed from decades-long preceding research activities represented, for instance, by Synge [1] in 1928 and Aloysuis [2] in 1956, who attempted to develop techniques for optical microscopes with ultra-high resolution.

Note that all of these attempts before 1980 and the majority of attempts after 1980 were based on the conventional theories of optics derived from Maxwell's theory of electromagnetism [3].

The vague but inspired image of the "small light field" held in the mind of M.O. was, however, more flexible in the sense that it may be a new kind of light field beyond the conventional framework of optical theory. As we can easily imagine, from the beginning, his ambitious attempts, including the discovery of this new class of light field, were snubbed by established leaders in the field who were, so to speak, the academic guardians of Maxwell's theory. His resolution was so unique in this particular point that no one else seemed to dare seek a similar research goal, but the original technology he later realized became quite distinguished. The first step towards his ambitious goal was to identify experimentally in more detail the existence

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of such a bizarre "small light field". Actually, he started from the *optical near-field*, which appears in the form of a membrane around a nanometre-sized particle under the irradiation of visible light, which he later renamed, focusing on its quantum mechanical nature, the *dressed photon* (DP) [4]: a metaphoric expression of photon energy fused partly with that of material involved in the light-matter field interaction.

In some studies, for instance [5], it seems that the evanescent light field is often regarded as a sort of optical near-field, as mentioned above, but, for the reason we give in the following section, we support the view that a DP, as a newly revised version of the optical near-field in our context of research pursuing nanometre-sized small light, is not evanescent light in the sense that the former is created by non-linear light-matter field interactions, while the latter is boundary-trapped linear waves, as in the case of Lamb waves in seismology or coastal Kelvin waves in oceanography. Figure 1 shows the typical cases of DP realization investigated thus far.

Employing trial and error approaches, the first theoretical endeavour to model the target enigmatic small light field was made by Ohtsu and Kobayashi (2004) [6] (work explained in this book but not reported in the form of a published paper). They investigated the possibility of representing the target field in terms of a new kind of exciton dynamics, where the *hypothetical nonresonant virtual polariton (HNVP)* plays a key role in the exciton. It is well known that there exist two different kinds of photon in a vacuum: a free photon and a virtual one that is a mediator of electromagnetic interactions. For electromagnetic field propagation through a given material, a

polariton can be regarded as (or corresponds to) the "free photon mode in a vacuum"; hence, it would be natural to assume the existence of the altered form of the virtual photon in the material. In their scenario, Ohtsu and Kobayashi tentatively introduced the above notion of an *HNVP* to represent this altered form of the virtual photon field. Here, we do not give the details of their analyses that employed this working hypothesis since they depended on some ad hoc assumptions to be improved. Guided by this working hypothesis, they somehow derived a formula describing a spherically symmetric localized distribution of the electromagnetic potential $A_0(r)$ of the small light field under consideration, presented as a Yukawa potential of the form:

$$A_0(r) = \frac{1}{r} \exp\left(-\frac{r}{a}\right), \tag{1}$$

where r and a respectively denote a radial coordinate and the radius of a given nanometre-sized particle.

Encouraged mainly by the success of experimental validation of the optical near-field and partly by the preliminary theoretical result of (1) together with the successful holding of an international symposium [3] in the early 1990s, M.O. hardened his conviction of the significance of his research and decided to disseminate it through society by establishing a new technical field named nanophotonics in the traditional optical society. This can be regarded as the start of the unique activity of nanophotonics [7]. The reason why we call it a unique activity is because the field of nanophotonics actively grew later to include such research themes as photonic crystals [8] and plasmonics
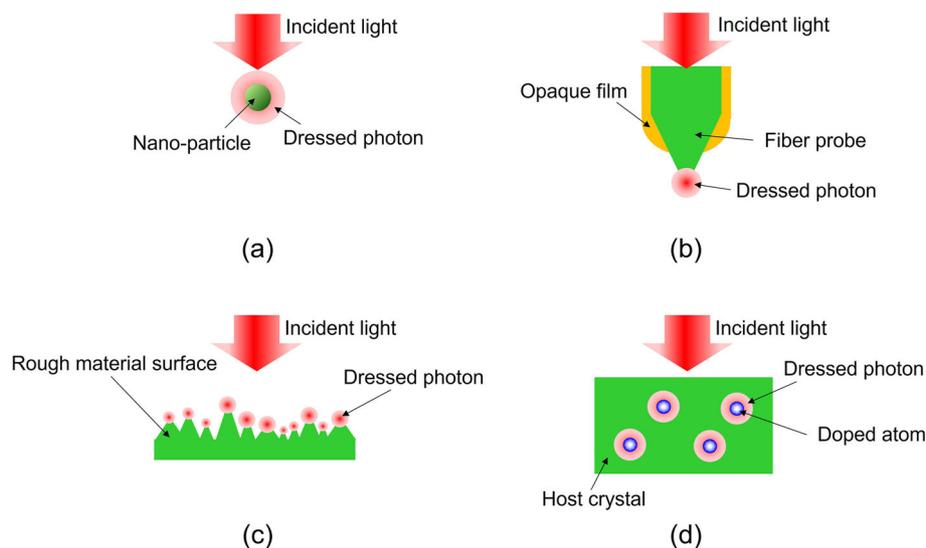

Fig. 1 Typical DP configurations. (**a**) On a nano-particle. (**b**) On the tip of a fiber probe. (**c**) On bumps of a rough material surface. (**d**) On doped atoms in a host crystal

[9]. It is clear that the light field employed in these studies is the conventional propagating light field, which has nothing to do with the optical near-field mentioned above.

By the end of the first decade of the 21st century, M.O.'s disseminating efforts had successfully born fruit in the form of distinguished national programs exerting considerable influence upon spectroscopic analysis, lithography, high-capacity optical storage, optical devices, autonomous smoothing of a material surface and optical information processing. Well-known readily recognizable feats achieved by DP technology include the case of light-emitting devices using indirect-transition-type silicon semiconductor crystals and autonomous smoothing of a material surface. The latest concise summary highlighting the unique DP technology, including the above-mentioned cases, was given in the introductory section of Sakuma et al. [10]. A comprehensible explanation of research activities on DPs during the above-mentioned first decade of the 21st century was given by Ohtsu et al. (2002) [7] and Ohtsu (2014) [4]. The former reference [7] showed that the term nanophotonics was first introduced officially by M.O.

In search of refinement of the theoretical models
On QFT with infinite degrees of freedom
A discontinuous jump in the theoretical study of DPs was brought about by the activities of a unique initiative launched in 2016 by M.O. To accelerate theoretical and modelling studies, he established a general incorporated association named Research Origin for Dressed Photon (RODreP), which operated as a virtual research laboratory connecting researchers in remote locations. On the basis of the recognition that a major impediment to developing DP theory is the lack of a satisfactory QFT, M.O. invited I. Ojima (the second author, hereafter I.O) as an expert in QFT, to join RODreP. Among I.O.'s achievements, his proposal of Micro-Macro duality (MMD) theory [11] has been remarked by M.O. as a modern version of *quantum-classical correspondence* proven in a mathematically rigorous fashion.

The important theoretical basis of MMD theory was prepared by the pertinent generalization of the notion of *superselection rule* consisting of *sectors* formulated originally by Doplicher-Haag-Roberts [12, 13]: in the context of DHR superselection theory, a *sector* is defined by a *factor* representation π of the observable algebra $\mathcal{A} = \mathcal{F}^G$ which is the fixed-point subalgebra of the field algebra $\mathcal{F}$ under a compact symmetry group G. Here a *factor* refers to the von Neumann algebra $\pi(\mathcal{A})''$ arising from π having a *trivial centre* $\mathfrak{Z}_\pi(\mathcal{A}) := \pi(\mathcal{A})' \cap \pi(\mathcal{A})'' = \mathbb{C}1$ and, at the level of field algebra $\mathcal{F}$, π yields an irreducible representation of G whose generators are called the *superselection charges*. According to this original DHR theory, sectors

have been understood only in discrete forms owing to the discreteness of $\hat{G}$ the dual of compact symmetry group G.

In the generalized formulation of sector theory due to [14, 16] applicable to wider contexts, the notion of *sectors* can simply be understood as the spectrum of the *centre* of (represented) observable algebra, either discrete or continuous depending on situations to be considered. Here *spectrum* means simultaneous eigenvalues due to the commutativity of the *centre* which admits the simultaneous diagonalization.

In the usual discussion, QM (quantum mechanics) and QFT (quantum field theory) are considered separately. Once they are treated in a unified way, according to which the former is just quantum system with *finite* degrees of freedom and the latter with *infinite* ones. From such a viewpoint the sharp contrast between QM and QFT becomes evident in the light of the Stone-von Neumann uniqueness theorem [15], whose validity in QM allows only one sector to exist for each finite QM system. Owing to this unicity, the variety of the macroscopic classical world cannot be explained as a result of emergence processes from the microscopic QM as described by I.O. [16]. In sharp contrast, QFT with infinite degrees of freedom can accommodate infinitely many sectors [14] which are mutually *disjoint* (refined notion of *unitary non-equivalence* sharply opposite to quantum coherence) whose nontrivial *centres* play roles of order parameters working as the commutative macroscopic physical quantities. In this way, the presence of multiple sectors in QFT with infinite degrees of freedom is seen to hold the key to MMD [11] as a general and universal version of *quantum-classical correspondence.*

An immediate and appealing consequence of MMD theory is that the longstanding problem of Schrödinger's cat is an ill-posed one in the sense that a cat as a macroscopic entity consisting of multiple sectors cannot be described properly by a quantum mechanical system possessing only one sector! In [17], Ojima stated: *. . . thus, such a common belief in quantum mechanics is wrong that any vector state given as a superposition $c_1\psi_1 + c_2\psi_2 + \cdots$ is a pure state showing quantum interference effects. From this viewpoint, the famous paradox of Schrödinger's cat is merely an ill-posed question, based on the level confusions about quantum-classical boundaries. Namely, because of the absence of such a physical observable A as $< \psi_{dead}|A\psi_{alive} > \neq 0$, the actual transition from the cat's being alive to dead can take place, not at the micro-level of the Geiger counter, but by macroscopic accumulation of infinitely many microscopic processes! This last point can be understood by such quantum-classical correspondence that classical macro level consisting of order parameters to describe inter-sectorial structure emerges from microscopic levels through condensation of infinite number of quanta. (As a matter of course, the presence or absence*

of microscopic observables triggering macroscopic state changes depends highly on the situations and/ or aspects in consideration, like the case, for instance, of the visual eyesight controlled by photo-chemical reactions involving the rhodopsin molecules at the retinae.)

To be precise, the absence of such observable A as $< \psi_{dead}|A\psi_{alive} > \neq 0$ in the above corresponds to the disjointness between the states of cat's being alive and dead in the system with infinite degrees of freedom, whose levels cannot be shifted to that of QM with finite degrees of freedom because of the absence of states corresponding to cat's being alive and dead. Namely, those states ψ_{alive} and ψ_{dead} of cat can exist only at the theoretical level of QFT where they are necessarily disjoint, and the usual QM type arguments frustrated by the formula $< \psi_{dead}|A\psi_{alive} > \neq 0$ cannot exist anywhere in the world.

Prevailing undue treatment of longitudinal waves

We do not think it a rare occasion that, say, in a standard text of electromagnetism, we find a sentence saying that an electromagnetic wave is not longitudinal but transversal. Presumably, the reason why such a statement is prevailing in a wide range of scientific communities is the influence of "advanced" quantum electrodynamics (QED) setting the trend of the new era of quantum technology, according to which the longitudinal modes are to be eliminated as unphysical ones. Nevertheless, the existence of classical longitudinal electromagnetic modes was reported unmistakably by Cicchitelli et al. [18] in an esteemed journal of Physical reviews. Thus, there seems to exist a discrepancy left unaddressed in the field of electromagnetism.

In our opinion, Ojima's MMD theory, which links the quantum world and the classical one, is quite helpful in resolving this discrepancy. In fact, Ojima [19] had already re-examined the processes of electromagnetic field quantization by employing the Nakanishi-Lautrup formalism [20] of manifestly covariant quantization. One of his important conclusions related to our present concern is that the mode eliminated by the conventional quantization remains to be physical as a *non-particle mode* and plays an important role in electromagnetic interactions associated with the longitudinal Coulomb mode. In the preceding subsection, we emphasized the existence of multiple generalized sectors in QFT, and now we see that Ojima's interpretation of the physicality of longitudinal modes is an important specific example of that without which we cannot have a consistent theory connecting micro- and macro-electromagnetism.

Concerning the Coulomb mode associated with the electromagnetic interaction, there exists an important piece of general knowledge that we have thus far ignored, that is, a mathematical criterion called the Greenberg-Robinson (GR) theorem [21, 22] that is used to distinguish non-linear field interactions from the free time evolutions of non-interacting modes. It states that *if the Fourier transform $\varphi(p)$ of a given quantum field $\varphi(x)$, where p and x respectively denote the momentum and position, does not contain an off-shell spacelike momentum p_μ with $p_\nu p^\nu < 0$, then $\varphi(x)$ is a generalized free field.* Although spacelike momenta are often associated with tachyons breaking the Einstein causality, it is known that there exist certain types of causal motions [23] having spacelike momenta. Hence, taking the above-mentioned GR theorem into consideration, we should not exclude certain dynamical modes simply because their momenta are spacelike.

Augmented spacelike Maxwell's equations

The arguments in the preceding subsection have revealed that the classical longitudinal mode reported by [18] must be closely related to virtual photons as the mediator of the longitudinal Coulomb force and that spacelike momenta must be considered in some form. At this point, it is worthwhile to point out that the introduction of spacelike (out of the light cone) characteristics of the electromagnetic potential A_μ is not new but is well known in the literature on the Aharanov-Bohm (AB) effect [24]. Namely, an observable quantity $\oint_\gamma A_\mu dl$ in the AB effect does not correspond to the value of A_μ at a certain point in spacetime but to the integrated one along the Wilson loop γ. Since detailed derivations of how new components of spacelike momentum can be brought into electromagnetism in a consistent fashion were already given by [10, 25], in the following, we show only some of the essential points of the full picture that are relevant to our present discussion on optics as a discipline of electromagnetism.

Now, consider the well-known Maxwell Eq. (2), represented in terms of the vector potential A^μ:

$$\partial_\nu F^{\mu\nu} = \partial_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) = [-\partial^\nu\partial_\nu A^\mu + \partial^\mu(\partial_\nu A^\nu)] = j^\mu. \tag{2}$$

Based on Helmholtz decomposition, A^μ can be decomposed into

$$A^\mu = \alpha^\mu + \partial^\mu\chi, \quad (\partial_\nu\alpha^\nu = 0, \quad \partial_\nu A^\nu = \partial_\nu\partial^\nu\chi). \tag{3}$$

The mixed form of energy-momentum tensor T_μ^ν associated with (2) becomes

$$T_\mu^{\ \nu} = -F_{\mu\kappa}F^{\nu\kappa} + \frac{1}{4}\eta_\mu^{\ \nu}F_{\alpha\beta}F^{\alpha\beta}, \tag{4}$$

where the notations are conventional and the sign convention of the Lorentzian metric ($\eta_{\mu\nu}$) signature $(+ - - -)$ is employed.

A well-known convention regarding A_μ is that, in (2), we can impose the Lorentz gauge condition $\partial_\nu A^\nu = 0$ since it is a non-physical quantity. As we will soon show, the Lorentz gauge condition can be relaxed without violating Maxwell's Eq. (2) once we accept that the longitudinal mode is physical. The aforementioned Nakanishi-Lautrup

formalism on the covariant quantization of the electromagnetic field actually permits a wide class of gauge conditions, among which the Feynman gauge condition of the form

$$\phi := \partial_\nu A^\nu, \quad \partial^\nu \partial_\nu \phi = 0, \tag{5}$$

is particularly important for our present discussion. Notice that the Feynman gauge permits us to regard $\partial_\nu A^\nu$ as a physical quantity ϕ (to be shown shortly) propagating in spacetime at the speed of light and to regard the Lorentz gauge condition as the ground state of ϕ, namely, $\phi = 0$.

To show the physicality of ϕ, let us consider the energy-momentum conservation law of $T_\mu{}^\nu$, which assumes the form $\partial_\nu T_\mu{}^\nu = 0$. Using (4) in the case of a free wave mode satisfying $F_{\sigma\tau}F^{\sigma\tau} = 0$, we obtain

$$\partial_\nu T_\mu{}^\nu = \partial_\nu(-F_{\mu\sigma}F^{\nu\sigma}) = F_{\mu\nu}\partial_\sigma F^{\nu\sigma} = F_{\mu\nu}j^\nu. \tag{6}$$

In (6), if the electric current j^ν vanishes, then we have $\partial_\nu T_\mu{}^\nu = 0$. However, there exists another case for which we also have $\partial_\nu T_\mu{}^\nu = 0$, that is,

$$\partial^\nu \partial_\nu A^\mu = 0, \quad \partial_\nu F^{\mu\nu} = \partial^\mu(\partial_\nu A^\nu) = \partial^\mu \phi \neq 0,$$
$$F_{\mu\nu}\partial_\sigma F^{\sigma\nu} = 0, \tag{7}$$

which can be directly derived from (2) and (7) under the condition of $F_{\mu\nu} \perp \partial^\nu \phi$ by saying that $\partial^\nu \phi$ is parallel to the Poynting 4-vector constructed from $F_{\mu\nu}$. Using (3), the first equation ($\partial^\nu \partial_\nu A^\mu = 0$) in (7) can be rewritten as

$$\partial^\nu \partial_\nu \alpha_\mu^{(h)} = 0, \quad \partial^\nu \partial_\nu \alpha_\mu^{(i)} + \partial^\nu \partial_\nu(\partial_\mu \chi) = 0, \tag{8}$$

where $\alpha_\mu^{(h)}$ and $\alpha_\mu^{(i)}$ respectively denote homogeneous and inhomogeneous solutions. Clearly, $\alpha_\mu^{(h)}$ represents a non-divergent transverse mode, while $\alpha_\mu^{(i)}$ is a particular solution to be specified by a given χ field that satisfies the second equation $[\partial^\nu \partial_\nu \phi = 0; (\phi = \partial^\nu \partial_\nu \chi)]$ in (5).

The second equation in (8) is considered as a balancing equation between a couple of rotational ($\alpha_\mu^{(h)}$) and irrotational ($\partial_\mu \chi$) fields. This balance is well documented in the case of two-dimensional (2d) irrotational motions of an incompressible fluid. The incompressibility of the fluid makes its motion non-divergent such that its 2d velocity field $(u(x,y), v(x,y))$ in Cartesian coordinates (x, y) is expressed by a stream function ψ, i.e., $u = -\partial_y \psi$, $v = \partial_x \psi$. On the other hand, because of the irrotationality of the velocity field, it can also be expressed by a velocity potential φ, i.e., $u = \partial_x \varphi$, $v = \partial_y \varphi$. Equating these two expressions, we obtain

$$u = -\partial_y \psi = \partial_x \varphi, \quad v = \partial_x \psi = \partial_y \varphi, \tag{9}$$

which is the well-known Cauchy-Riemann relation in complex analyses. Utilizing this knowledge in fluid dynamics, Sakuma et al. [26] showed that the balance equation in (8) enables us to interpret *the current $\partial^\mu \phi$*

introduced in the second equation in (7) as the longitudinally propagating electric field.

With the introduction of this defined physical quantity ϕ, we can augment the conventional Maxwell equation such that its augmented form naturally includes an additional spacelike momentum branch. Since the conventional electromagnetic wave field consists of propagating electric and magnetic fields that are perpendicular to each other, the classical counterpart model of the virtual photon we seek would also consist of $\partial^\mu \phi$ and a certain gradient vector field perpendicular to it. Notice that a vector field that is perpendicular to the lightlike vector $\partial^\mu \phi$ is either a spacelike one or $\partial^\mu \phi$ itself, which leads us to assume that the other gradient vector field $\partial_\mu \lambda$ must satisfy a spacelike Klein-Gordon (KG) equation of the form

$$\partial^\nu \partial_\nu \lambda - (\kappa_0)^2 \lambda = 0, \tag{10}$$

where κ_0 denotes a certain constant to be determined later. The fact that the vector field $\partial_\mu \lambda$ has spacelike characteristics seems to be an advantageous factor in our construction of the spacelike momentum branch of Maxwell's equation based on the consequence of GR theorem. Actually, this conjecture turns out to be the case, and Sakuma et al. [10] successfully formulated the targeted spacelike momentum branch of Maxwell's equation, which is called the Clebsch dual (CD) electromagnetic field because, in formulating the theory, the Clebsch parameterization was employed to express the spacelike electromagnetic 4-vector potential U_μ in terms of ϕ and λ. As shown below, there exist two different types of CD field, namely, a lightlike field (case [**I**]) possessing the properties of the gauge as well as conformal symmetries and a spacelike field (case [**II**]) for which the above-mentioned symmetries break.

[**I**] For the lightlike case of $U^\nu(U_\nu)^* = 0$, where $(U_\nu)^*$ denotes the complex conjugate of U_ν, by the introduction of new vector notations:

$$C_\mu := \partial_\mu \phi, \quad (\partial^\nu \partial_\nu \phi = 0); \quad L_\mu := \partial_\mu \lambda,$$
$$(\partial^\nu \partial_\nu \lambda - (\kappa_0)^2 \lambda = 0), \tag{11}$$

the key equations of the CD field become

$$U_\mu := \lambda \partial_\mu \phi, \quad U^\nu \partial_\nu U_\mu = 0, \quad \partial^\nu \partial_\nu U_\mu - (\kappa_0)^2 U_\mu = 0, \tag{12}$$

$$S_{\mu\nu} := \partial_\mu U_\nu - \partial_\nu U_\mu = L_\mu C_\nu - L_\nu C_\mu, \quad C^\nu L_\nu = 0. \tag{13}$$

Moreover, the associated energy-momentum field $\hat{T}_\mu{}^\nu$ assumes the form

$$\hat{T}_\mu{}^\nu = S_{\mu\sigma}S^{\nu\sigma} = \rho C_\mu C^\nu, \quad \rho := L^\nu L_\nu \left[L^\nu (L_\nu)^* < 0 \right];$$
$$\partial_\nu \hat{T}_\mu{}^\nu = 0. \tag{14}$$

[II] For the spacelike case of $U^\nu(U_\nu)^* < 0$, with the modified definitions of

$$C_\mu := \partial_\mu\phi, \quad (\partial^\nu\partial_\nu\phi - (\kappa_0)^2\phi = 0);$$
$$L_\mu := \partial_\mu\lambda, \quad (\partial^\nu\partial_\nu\lambda - (\kappa_0)^2\lambda = 0), \quad (15)$$

we have

$$U_\mu := \frac{1}{2}(\lambda C_\mu - \phi L_\mu); \quad U^\nu\partial_\nu U_\mu = 0, \quad (16)$$

and (13) remains valid in this extended case. The associated energy-momentum field $\hat{T}_\mu^\nu$ now becomes

$$\hat{T}_\mu^\nu = \hat{S}_{\mu\sigma}{}^{\nu\sigma} - \frac{1}{2}\hat{S}_{\alpha\beta}{}^{\alpha\beta}\eta_\mu^\nu, \quad \hat{S}_{\alpha\beta\gamma\delta} := S_{\alpha\beta}S_{\gamma\delta}. \quad (17)$$

The gauge invariance of the electromagnetic field is directly related to the conservation of j^μ, which is readily shown from (2), namely, $\partial_\mu j^\mu = \partial_\mu\partial_\nu F^{\mu\nu} = 0$. In our case of $\partial_\nu F^{\mu\nu} = \partial^\mu\phi$ given by the second equation of (7) in lightlike case [I], the gauge invariance condition $\partial_\mu\partial_\nu F^{\mu\nu} = 0$ becomes equal to the second equation $(\partial^\mu\partial_\mu\phi = 0)$ in (5). Thus, we see that the well-known statement that the gauge condition prevents a photon from having mass can be applied to case [I] of the CD field. In addition, from the first equation in (14), we see that the particle form of $\hat{T}_\mu^\nu = \rho C_\mu C^\nu$ is unphysical in the sense that ρ becomes negative, while the alternative wave form of $\hat{T}_\mu^\nu = S_{\mu\sigma}S^{\nu\sigma}$ does not seem to cause any problem, which is consistent with the aforementioned statement of Ojima that the longitudinal "virtual photon mode" becomes physical in a non-particle mode.

Case [II] can be regarded as a gauge symmetry-breaking and conformal symmetry-breaking case. In [10], we show that $\hat{S}_{\alpha\beta\gamma\delta}$ has exactly the same symmetry as the Riemann curvature tensor $R_{\alpha\beta\gamma\delta}$ and that the results in case [II] can be extended to a curved spacetime such that $\hat{T}_\mu^\nu$ in (17) is isomorphic to the Einstein tensor in general relativity. This fact tells us that the spacelike energy-momentum tensor $\hat{T}_\mu^\nu$ of the CD field naturally fits into the general theory of relativity and that the CD formulation can remain valid for cosmological problems.

One of the intriguing expressions in the CD formulation is Eq. (14), which shows that the energy-momentum tensor $\hat{T}_\mu^\nu$ has dual expressions corresponding to the wavelike field $S_{\mu\sigma}S^{\nu\sigma}$ and to the particle-like field $\rho C_\mu C^\nu$, the latter of which appears similar to Veronese embedding in projective geometry. From this viewpoint, Ochiai [27] discussed the symmetry of the DP model using the CD formulation and showed that the symmetry is well described in terms of the compact homogeneous space, such as Grassmann and flag manifolds with arbitrary dimensions. In "Observed "photon cluster" and light field with spin zero" section on the novel experimental discovery of DP light, we return to Eq. (14) for its theoretical implication.

New dynamical model of DPs

Before proceeding to the formulation of the new DP model, we point out an important fact: the quantized field corresponding to the spacelike KG Eq. (10) is given by the Majorana field. To support this, let us consider the Dirac equation

$$(i\gamma^\nu\partial_\nu + m)\Psi = 0, \quad (18)$$

which can be regarded as the "square root" of the timelike KG equation, i.e., $(\partial^\nu\partial_\nu + m^2)\Psi = 0$, from which we can readily see that the Dirac equation for $(\partial^\nu\partial_\nu - (\kappa_0)^2)\Psi = 0$ must be $i(\gamma^\nu\partial_\nu + \kappa_0)\Psi = 0$. It is reported that for (18), there exists an electrically neutral Majorana representation in which all the values of the γ matrix become purely imaginary numbers such that it takes the form $[(\gamma_{(M)})^\nu\partial_\nu + m]\Psi = 0$, which clearly shows that a Majorana field λ satisfying $(\gamma^\nu\partial_\nu + \kappa_0)\lambda = 0$ corresponds to the spacelike KG Eq. (10). In [10], the mechanism of how a couple of fermionic Majorana fields λ and ϕ with a half-integer spin 1/2 can form a bosonic field with spin 1 corresponding to the CD field is explained based on Pauli's exclusion principle reflected in the orthogonal condition $C^\nu L_\nu = 0$ in (13). Another important aspect of the quantization revealed by the study of [10] is that the length (or wavenumber) must be quantized in the Majorana field, which seems to correspond to the successful derivation of spacetime quantization performed by Snyder [28], who worked on the spacelike momentum field defined on de Sitter space. We explain the advantage of wavenumber quantization over length quantization shortly.

As we explained in Fig. 1, the DP field appears around a point-like singularity. Thus, as the simplest toy model for the DP generation mechanism, let us consider a case in which a spacelike field λ satisfying (10) is perturbed by the interaction with a point source of the form $\delta(x^0)\delta(r)$, where x^0 and r respectively denote time and a radial coordinate of a spherical coordinate system. This system was already studied by Aharonov et al. [23], who showed that the resulting behaviour of the solution can be expressed by the superposition of a spacelike stable oscillatory mode and a timelike linearly unstable mode whose combined amplitude moves at a speed less than the light velocity. A timelike unstable mode of the solution to the perturbed (10) expressed in polar coordinates has the form $\lambda(x^0, r) = \exp(\pm k_0 x^0)R(r)$, where $R(r)$ satisfies

$$R'' + \frac{2}{r}R' - (\hat{\kappa}_r)^2 R = 0, \quad (\hat{\kappa}_r)^2 := (k_0)^2 - (\kappa_0)^2 > 0. \quad (19)$$

The solution $R(r)$ is known as the Yukawa potential (cf. (1)), of the form

$$R(r) = \frac{1}{r}\exp(-\hat{\kappa}_r r), \quad (20)$$

which rapidly falls off as r increases.

Quantum mechanically, since k_0 is the energy of a given system, taking into account the aforementioned quantization in the Majorana field, the wavenumber quantization with $Min[k_0] = \kappa_0$ and with $\Delta k_0 = \kappa_0$ seems to be advantageous over the length quantization because the former is similar to the well-known energy quantization of $E = h\nu$. Hence, we assume that this wavenumber quantization is valid for the quantum version of the second equation in (19). Another crucial kinematic property that distinguishes quantum mechanics from classical mechanics is the temporal directions of a moving particle and anti-particle. For an electrically neutral Majorana particle field, this property corresponds directly to the time reversal, which means that a couple of unstable fields $\lambda(x^0, r) = \exp(\pm k_0 x^0) R(r)$ in the classical system can be reinterpreted as a particle versus anti-particle pair in a quantum mechanical system. Thus, the above argument shows that as a result of field interactions between (10) and $\delta(x^0)\delta(r)$, a pair consisting of a timelike Majorana particle and an anti-particle pop up at the origin $r = 0$. However, those particle fields are non-propagating; hence, the mechanism of pair annihilation would occur instantly to produce a "small light field" with a spatial distribution $R(r)$. We believe this to be a basic mechanism of DP genesis. In the case of a pair annihilation having anti-parallel spins, the resulting spin 0 DP has an electric nature, while in the case of parallel spins ± 1, the DP has a magnetic nature. We conjecture that such a magnetic DP may exist and is involved in the large magneto-optical effect boosted by the DP, as recently reported by Kadowaki et al. [29].

The advantage of the newly derived radial distribution given by $R(r)$ in (20) over $A_0(r)$ given by Eq. (1) discussed in the introductory section is the fact that the former distribution $R(r)$ has a clear-cut minimum value κ_0 of $\hat{\kappa}_r$, while for $A_0(r)$, the wavenumber is determined solely by the inverse of the radius of a given nanosized particle. The existence of the minimum wavenumber κ_0 means that there exists a maximum size of the DP defined by

$$L_{max}^{(dp)} := \frac{1}{\kappa_0} \approx (50 - 70) nm, \qquad (21)$$

whose magnitude was derived experimentally by Ohtsu and Kawazoe [30, 31]; here, we call the $L_{max}^{(dp)}$ DP constant. From the viewpoint of *a new natural unit system* in which all the magnitudes of the Planck constant h, light velocity c and $L_{max}^{(dp)}$ are set to unity, the importance of this length was discussed by Sakuma and Ojima [25]. They showed that it gives the geometrical mean of the smallest Planck length and the largest length associated with a newly modified cosmological constant, related to their dark energy model defined by the "ground state" of a spacelike Majorana field and to their novel dark matter model defined solely by the Weyl conformal tensor field.

In the above arguments developed in this subsection, we see that the small spatial structure of DPs originates from a deformed transition of the spacelike momentum of the Majorana field into a timelike one, in which process we may regard the DP that arises as a new type of "exciton" created by the irradiation of light at singular points. We have already pointed out that the second equation in (19) can be quantized such that $Min[k_0] = \kappa_0$ with $\Delta k_0 = \kappa_0$, which suggests that thus quantized $\hat{\kappa}_r$ can be regarded as the quantized energy level of the DP as an exciton. One of the important excitons in semiconductor physics is a bosonic electron-hole-pair, whose Bohr radius is approximately 10 nm. Notice that the DP, as a bosonic exciton, has a couple of properties similar to those of the electron-hole-pair, namely, its pair structure and size. For DPs, thus generated or annihilated around point-like singularities in a given material (a mathematically simplified model of a dopant), a phonon field, as the quantized lattice oscillations, works as their dynamical environment; thus, the goal of DP dynamics would describe DPs' behaviours, i.e., coupling with the environmental phonon field, with their "entries" and "outlets" at singular locations. Since a DP has a highly localized spatial structure, several researchers collaborating with RODreP, for instance, Hamano and Saigo [32], Konno et al. [33] and Higuchi et al. [34], are now developing models that describe DP behaviours by employing quantum walk approaches.

Observed "photon cluster" and light field with spin zero

Crystalline silicon had long been considered unsuited for light-emitting devices such as LEDs and lasers because it is a typical indirect-transition-type semiconductor. As mentioned at the end of the introduction, this common-sense view was already demolished by a series of experiments on DPs [4, 35], which showed that, through DP—phonon (DPP) annealing, the rearrangement of given doped atoms (see case (d) in Fig. 1) occurs such that it changes silicon into an optically active substance. To understand this unexpected and aberrational phenomenon, we have thus far carried out preliminary studies using, for instance, a working hypothesis such as the HNVP referred to in the introduction. In the last paragraph of the preceding section, however, we introduced an entirely new view on the DP as an exciton that resembles an electron-hole-pair in some respects. It seems that this new view can provide a straightforward explanation for the enhanced optical activity of silicon in terms of the exciton dynamics of DPs, which is basically free from the different band gap structures between direct- and indirect-transition-type semiconductors.

Specifically, the new theory of DPs explained in two sequencing sections of "Augmented spacelike Maxwell's equations" and "New dynamical model of DPs" suggests

the possibility (to be shown shortly) that the light emitted from DP-involved mechanisms can be a unique light field with spin zero, which behaves as a particle. Clearly, the conventional theory of light does not cover such a light field because it deals only with *on-shell* transverse modes. The reason why a light field with spin zero behaves as a particle is due to the following theorem proved by Wightman [36] stating that

a Lorentz or Galilei covariant massive system is always localizable. For the Lorentz case, the only localizable massless elementary system (i.e., irreducible representation) has spin zero,

where localizability means that we can define a position operator for that field. In section of "Augmented spacelike Maxwell's equations", we showed that lightlike case [**I**] of the CD field can be described by the system of Eqs. (11)–(14), in which the field strength $S_{\mu\nu}$ is expressed in terms of bivectors C^{ν} and L_{ν}, which satisfy the orthogonality condition $C^{\nu}L_{\nu} = 0$. In case [**I**], we assume that L_{ν} is a spacelike vector. Notice, however, that the orthogonality condition $C^{\nu}L_{\nu} = 0$ is also satisfied in the case of $L_{\nu} = C_{\nu}$ since C_{ν} is a null vector. Of course, in that case, the vortical field strength $S_{\mu\nu}$ vanishes. Recall that, quantum mechanically, the C_{ν} field is a Majorana field with spin $1/2$; thus, a couple of anti-parallel C_{ν} fields with spin $1/2$ and $-1/2$ can be combined to yield a null energy-momentum current $C_{\mu}C^{\nu}$ (cf. $\hat{T}_{\mu}{}^{\nu} = \rho C_{\mu}C^{\nu}$ in (14)) with spin zero, which can be regarded as unique bullet-like light field with spin zero.

Though we have been looking experimentally into the emission spectra and intensity of luminous phenomena of silicon, few efforts have been made to consider the temporal behaviours of emissions expressed in terms of *the second-order (intensity) cross-correlation coefficient (the 2nd-order CC)*, especially for behaviours in response to the injected current. The main reason for that is because there exist few infrared light sensor devices suitable for temporal resolution measurements applied to the emission band spanning the wavelength range of $(1.1–2.0)$ μm. In the following, as a cutting-edge flash report, we give the outline and the main result of our ongoing experiment related closely to the above-mentioned temporal behaviours of emissions as well as to the prediction of our new DP model. As the background information of the present experiment, we first briefly refer to a preceding experiment conducted by Wada et al. [37]. They showed that a silicon-LED with an emission band of $(1.3–1.6)$ μm, successfully fabricated by our new technology using DPs, works as a relaxation oscillator upon the injection of direct current, yielding a pulse train of emission.

As a refined version of this experiment, most recently, the 2nd-order CC was evaluated much more precisely by Kawazoe et al. [38]. The basic setting of this experiment was that of the well-known Hanbury Brown-Twiss

method [39], the details of which are reported in full elsewhere. In fact, we already performed a similar experiment [40] in which we checked the behaviour of a single photon in a nanometre-sized semiconductor logic gate whose signal processing is carried out by using DPs. This time, to attain higher measurement accuracy, we employed a highly sensitive superconducting single-photon detector enabling us to measure the temporal behaviour of emission from a small luminescent spot on the surface of a given LED, which takes the form of a pulse train whose duration and repetition frequency are respectively approximately 50 ps and 1 GHz.

Figure 2 shows the value of the CC evaluated by our low noise Hanbury Brown-Twiss experimental setup. It represents two features: One is that the value of CC is smaller than unity in the range of time difference $|\tau| < 20$ ns. It indicates the photon antibunching (PA) phenomenon that is an inherent feature of a single photon. The other is that the CC takes a nonzero value at $\tau = 0$ even though it is smaller than 1×10^{-2}. This small nonzero value is attributed to the photons generated from multiple luminescent spots located in close proximity with each other in the LED surface. If we interpret these two features, it suggests a possibility that a cluster of "photon" emitted in our DP-involved experiment behaves approximately as if it is a single photon. Let us tentatively call it DP-cluster light (DP-CL). At present, we do not know the exact reason why this DP-CL exists, but we conjecture that it must be closely related to the localizable property of the spin zero particle we pointed out in relation to the Wightman theorem. Namely, if the observable positions of given spin zero quantum particles are "sufficiently close", then we believe that the cluster of those particles would behave as if it is a single quantum particle with the accumulated amount of energy.

Fig. 2 The second-order intensity CC for the Hanbury Brown-Twiss experiment
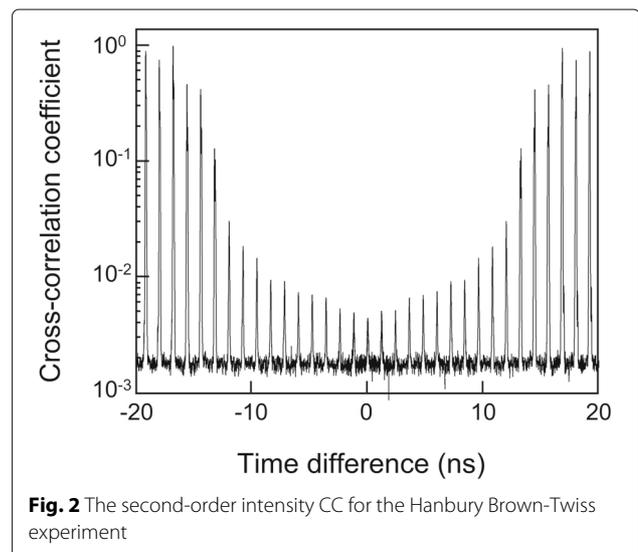
Future perspective on the new activities

In this article, we have explained a novel theoretical endeavour to develop a DP model in terms of the Majorana field and reported an unexpected finding of a novel kind of light that does not have a wavelike representation. As we referred to in the introductory section, the original nanophotonic technology employing DPs has achieved a variety of breakthroughs to date, for instance, in lithography, large-capacity optical storage, optical devices and fine surface smoothing. By virtue of the new theory, we now have a deeper understanding of the creation as well as annihilation mechanism of DPs as the new kind of excitons, of which the latter mechanism must be investigated further from the viewpoint of quantum DP light discussed in the preceding section. In addition, the new theory has revealed that, as briefly mentioned at the end of "New dynamical model of DPs" suggests the essence of DP dynamics is to describe the behaviours of "DP-excitons" affected by the environmental phonon field arising from the lattice vibrations of a given material. Thus, from the viewpoint of engineering, we can say that, among others, the acceleration of the development of quantum walk models capable of predicting the behaviours of "DP-excitons" is a pressing issue.

As extended fundamental research themes related to the DP constant given in (21), Sakuma et al. [10, 25] pointed out that we can cast new light on a series of the following cosmological unsolved issues, which must be important new directions in the research of the Majorana field:

[1] On dark energy

Dark energy is one of the cosmological enigmas that was introduced to explain observed cosmic accelerated expansion. We can safely say that we do not have a credible model for dark energy. Usually, it is modelled by the cosmological term of the form $\lambda g_{\mu\nu}$, where λ and $g_{\mu\nu}$ respectively denote the cosmological constant and metric tensor. First, note that the physical meaning of $\lambda g_{\mu\nu}$ has remained an unsolved issue since the time of Einstein (it will be covered in the subsequent item *[2]*). Intuitively, one may believe that the difference between the two scales of DPs and the cosmological constant λ is so extremely large that it would be irrelevant to link DPs with $\lambda g_{\mu\nu}$. In our opinion, a simple justification for this comparison is that in four-dimensional spacetime, a source-free Maxwell equation is conformally symmetric, and de Sitter space, as a solution to the Einstein equation corresponding to $\lambda g_{\mu\nu}$, exhibits scale-independent self-similarity. Thus, the scale difference does not matter in our comparison of these two. Actually, Sakuma et al. showed that although de Sitter space is related to dark energy, there exists a to date unnoticed possibility that dark energy can be related not to $\lambda g_{\mu\nu}$ but to the existence of a unique compound state of a Majorana field having spin 3/2 that accompanies *a reduced form of the cosmological constant* λ. They showed that the

theoretically estimated value of the reduced cosmological constant $\lambda_{(de)}$ based on their model is $2.47 \times 10^{-53} m^{-2}$, while the observationally derived (by the Planck satellite) $\lambda_{(obs)}$ is $3.7 \times 10^{-53} m^{-2}$.

[2] On the physical meaning of the cosmological term $\lambda g_{\mu\nu}$ and dark matter

Based on their above-mentioned analyses, Sakuma et al. further pointed out important possibilities. First, contrary to the prevailing conjecture that $\lambda g_{\mu\nu}$ represents the vacuum energy, which is related to what is known as *the worst theoretical prediction in the history of physics*, they claimed that it can be interpreted as the energy-momentum field of dark matter if we specify λ such that

$$\lambda = \lambda_{(dm)} := -\frac{1}{3}\lambda_{(de)}, \tag{22}$$

where $\lambda_{(dm)}$ denotes a newly introduced "modified cosmological constant" for which the factor 1/3 corresponds to the observed abundance ratio of dark matter over dark energy and the minus sign in front of this factor indicates the attractive nature of the gravitational force dark matter would possess. A new conjecture that $\lambda_{(dm)}g_{\mu\nu}$ must be a special form of gravitational energy-momentum field can be shown directly by rewriting $g_{\mu\nu}$ solely in terms of the Weyl curvature tensor $W_{\alpha\beta\gamma\delta}$ by using the following identity:

$$W_{\mu\alpha\beta\gamma}W_\nu^{\alpha\beta\gamma} - \frac{1}{4}W^2 g_{\mu\nu} = 0, \quad W^2 := W_{\alpha\beta\gamma\delta}W^{\alpha\beta\gamma\delta}. \tag{23}$$

For a simple spherically symmetric Schwarzschild solution of a given star, W^2 is a monotonically decreasing function; thus, the assumption of $W^2 \neq 0$ seems to be an acceptable condition for problems with galactic scale unless we consider these phenomena as gravitational waves.

[3] On the conformal cyclic cosmology of twin universes

In item *[1]*, we discussed de Sitter space in connection with dark energy. It is known that de Sitter space has a unique structural characteristic of twin universes, each of which is separated by a hyper-surface of the event horizon embedded in it. Interestingly, it is shown in [25] that the conformal cyclic cosmology (CCC) [41] proposed by Penrose may be combined with the twin-universe structure. The advantages of this proposal in cosmology are twofold. First, it would naturally resolve the problem of missing anti-matter that exists in the counterpart universe. Second, the genesis of those twin universes can be described in terms of a cyclic scenario similar to CCC, in which the birth and death of the twin universes can be compared to the creation and annihilation of a pair of elementary particles through the intervention of conformally symmetric light fields.

[4] Further research on DP-CL

At the beginning of the introduction, we pointed out that the original motivation of our DP study was to generate a small light field free from the diffraction. We believe that the preliminary flash report in the section of "Observed "photon cluster" and light field with spin zero" suggests the existence of such a peculiar propagating light field whose energy-momentum tensor assumes exactly the same form as the one of free fluid particles. If that is the case, then a laser beam consisting of this light field must be free from the diffraction since it behaves as a bullet. It is well known that a laser cannon employing the conventional transverse light waves suffers from a certain unavoidable restrictive condition depending on the beam diameter, range, spot diameter and wavelength, which arises from the diffraction of waves. It is true that there exists a certain class of diffraction-free mode-solutions [42] for transverse light waves, but we should not confuse those solutions with the above-mentioned peculiar light field intrinsically free from the diffraction. In regard to the peculiar light field, we further conjecture that the mechanism of DP-CL may be involved in γ-ray bursts, again one of the cosmological enigmas, as an intermittent extremely high-energy radiation with strong directionality reaching us after travelling over an enormous distance of several billions of light years.

Finally, as the concluding remark of this article, we comment on a new term, i.e., "off-shell science", the symbolic term M.O. introduced to disseminate the new field of science that DP studies have opened up. At the turn of the 20th century, a novel notion of the *light energy quantum* introduced by Max Planck expanded the frontier of physical sciences to include the microscopic world, with quantum mechanics taking the place of classical physics and the Planck constant h becoming the icon of new physics. As we have explained in this article, the new theory on DPs depends essentially on the extension of the momentum domain to include spacelike momenta outside of light cones, and hence, they are off-shell quantities from the viewpoint of conventional physics, which mainly focuses on timelike momenta constrained on given mass shells parameterized by the rest mass of the particles under consideration (the generalized expression of the most well-known equation: $E = mc^2$). In this final section, we saw only that the consequence of this extension of momentum is profound in the sense that it affects a large disciplinary area of physical science all the way down from the largest cosmology to the smallest particle physics. The DP constant given in (21) now stands as the icon of the emerging new off-shell science covering a large area of science, as was the case of h for quantum mechanics.

Abbreviations
AB effect: (Aharanov-Bohm effect); CC: (Cross-correlation coefficient); CCC: (Conformal cyclic cosmology); CD: (Clebsch dual); DP: (Dressed photon); DP-CL: (DP-cluster light); DPP: (DP—phonon); GR: (Greenberg-Robinson); HNVP: (Hypothetical nonresonant virtual polariton); KG: (Klein-Gordon); LED: (Light-emitting diode); MMD: (Micro-macro duality); PA: (Photon antibunching); QED: (Quantum electrodynamics); QFT: (Quantum field theory); QM: (Quantum mechanics); RODreP: (Research Origin for Dressed Photon)

Declarations

Competing interests
The authors declare that they have no competing interests.

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